Bulletin 37

Estimating Sustainable Yield to a Well in Heterogeneous Strata

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Alberta
RESEARCH COUNCIL

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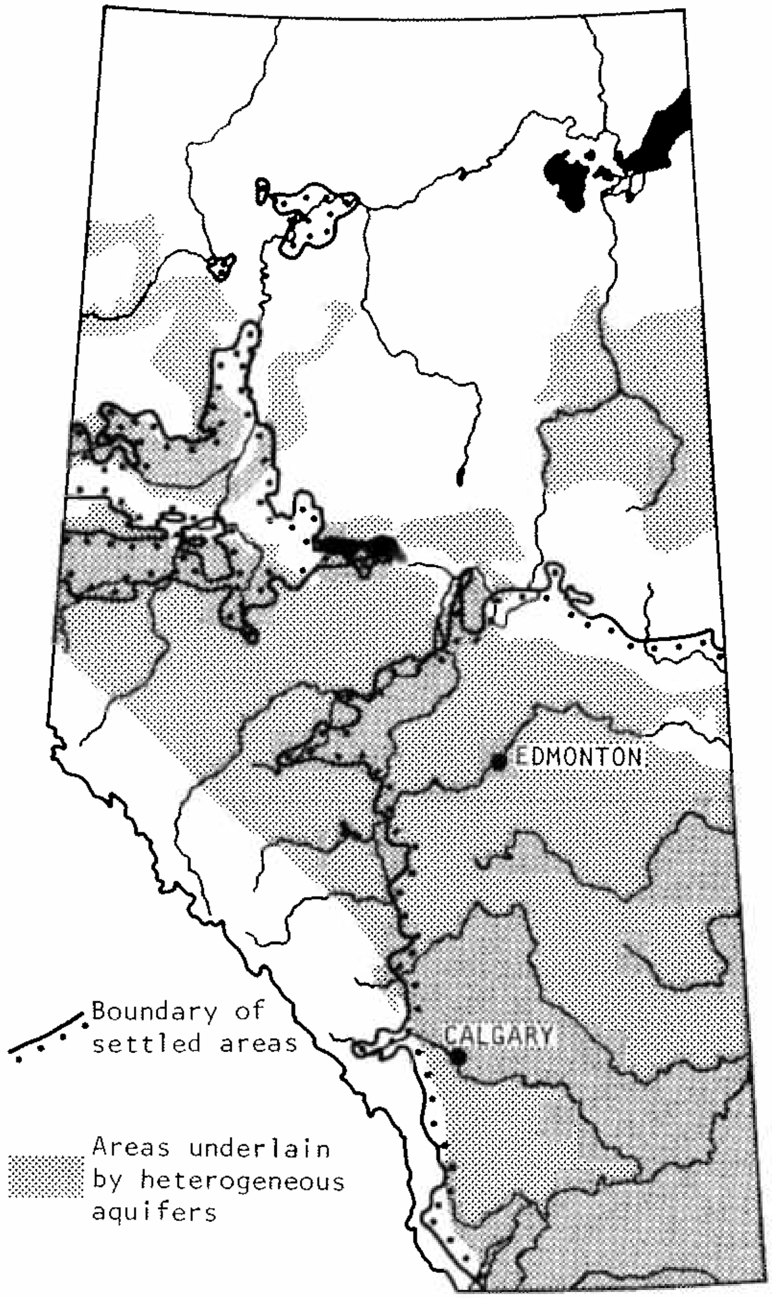

Figure 1. Settled areas and areas of heterogeneous bedrock aquifers, Alberta

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Estimating Sustainable Yield to a Well in Heterogeneous Strata

ABSTRACT

In Alberta, aquifers are heterogeneous, so current methods of predicting sustainable yields are inappropriate. Even the common term "transmissivity" is not applicable, and should be replaced by the new term "transmissive capacity", which is defined so as to take into account the dependence on time of the average conductive capability of the volume drained by a well during production. Estimators based on a sample of short-term transmissive capacities can be used to predict the expected value of long-term transmissive capacity at the producing well. Once estimates of long-term transmissive capacity are available, both 20-year safe yield and the drawdown at the well resulting from any production program can be calculated. The method suggested for estimating long-term transmissive capacity is empirical, so must be validated by successful application to suitable field cases. This method was applied to two field cases in Alberta, but further validation is required. However, the method presented for calculating safe yield depends only on the value of the long-term transmissive capacity, not on how the value was determined, so it can be used whenever that information is available.

INTRODUCTION

The physics of groundwater flow has been extensively investigated, making it possible to give a reasonable mathematical description of the processes involved. This description usually devolves to a partial differential equation and associated boundary conditions, because the groundwater system is usually considered to be a distributed-parameter system. Further, the application of numerical techniques, using high-speed digital computers, has facilitated the approximate solution of these partial differential equations, so that the behavior of a groundwater system in almost any circumstance can be modeled. It is reasonable to conclude, therefore, that predicting the behavior of groundwater systems has become little more than a formality. However, a mandatory input into a numerical model is the spatial variation of the system's hydraulic parameters. This information is, in general, not available, and in heterogeneous systems cannot be satisfactorily approximated. Thus, the current situation is that methods capable of predicting heterogeneous system behavior are available if heterogeneity is known, but no means exist for specifying the heterogeneity itself.

A specific example of this general dilemma is the prediction of long-term well performance in heterogeneous aquifers. There exist no practical methods for determining the spatial variations of transmissivity of heterogeneous aquifers. Consequently, it is impossible to predict the changes in water levels that will result from some arbitrary withdrawal program either at the production well or at any point in the aquifer.

This situation is very common in Alberta, where 45 percent by surface area (115,000 sq mi) of the bedrock aquifers having potable groundwater are heterogeneous. This area includes 75 percent of the settled parts of the province (Fig. 1). In these settled areas almost all of the farms and many of the towns use a groundwater source for water supply.

HYDROGEOLOGICAL SETTING

The heterogeneous aquifers in Alberta are Cretaceous and Tertiary continental clastic sandstones, shales, coals, and siltstones. Facies changes may be rapid both laterally and vertically. Figure 2 shows a schematic profile of the Wapiti Formation in the Edmonton area. Although the scale is large, with electric logs available only at intervals of approximately 1 mi, it is hardly possible to recognize any distinct lithologic horizons, except perhaps the basal sandstone overlying the Lea Park marine shales. As shown in the profile, sandstone lenses and coal seams of limited extent are embedded in a siltstone-shale matrix. Some zones have a relatively large number of sandstone lenses or coal seams, while other zones are predominantly shaly. Areally the sandstones have the appearance of channels. Adding further to the degree of heterogeneity are fractured zones, usually in the coal and shale and especially near land surface.

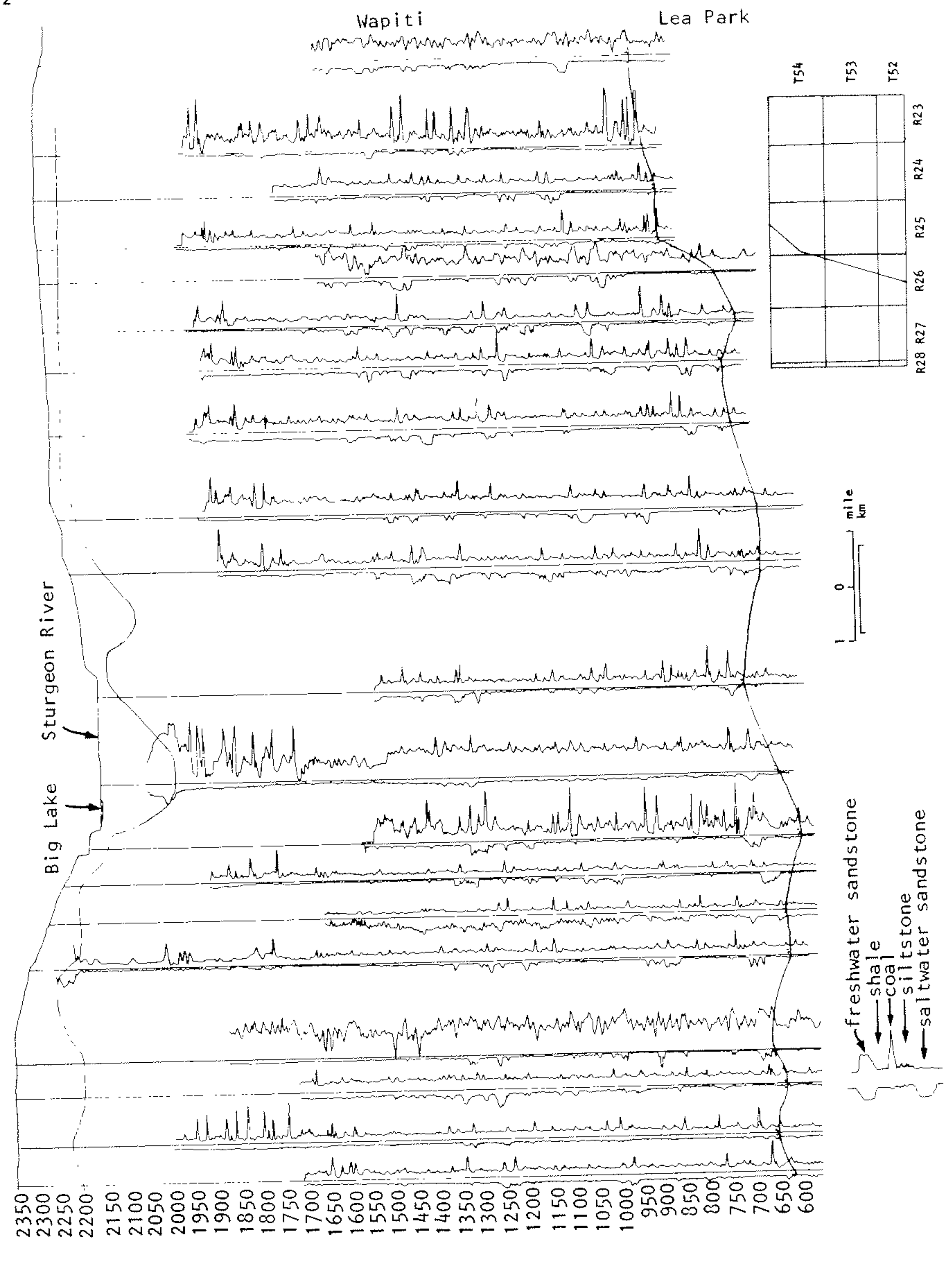


Figure 2. Profile of the Wapiti Formation

The degree of heterogeneity is also clearly demonstrated by the very wide range of transmissivity values in small areas. For example, in an area of 4 sq mi near Olds, 33 pump tests gave transmissivities ranging from 27 to 58,000 igpd/ft.

Permeability in this heterogeneous rock mass is anisotropic in varying degrees, but all parts have permeabilities significantly different from zero. Thus all rock units are capable of transmitting water; there are no truly confined aquifers in the sense of a permeable zone bounded by rocks of effective zero permeability, even though water entering the high-permeability lenses and channels must pass through the low-permeability shales.

The whole rock mass is fully saturated below the water table and, due to its non-zero permeability, there is complete hydraulic continuity. The water table has a controlling role in this hydraulic regime. It closely resembles the surface topography and is essentially the only boundary to the saturated zone, which effectively extends without limit downward and laterally.

However, the hydraulic system, despite its being unconfined, does not respond to pumping like a typical water table system. Because the open interval is separated from the water table by an interval of low permeability matrix, there is usually little observable drainage of the strata and only a small drop in water table during a pump test of several days' length. In general, during the early stages of pumping, water is released due to its own compressibility and that of the rock mass, a condition more characteristic of truly confined aquifers. Over the long term, however, the contribution from drainage of strata predominates.

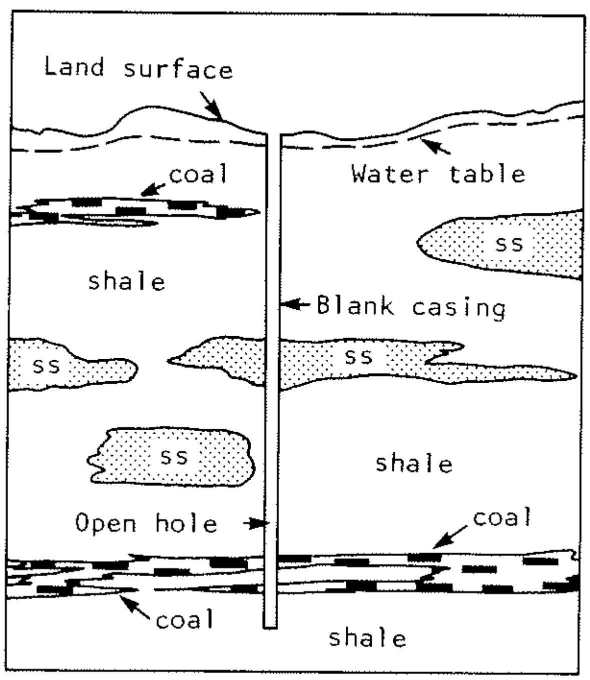


Figure 3. Typical well completion in an heterogeneous setting

A typical water-supply well in this hydrogeological setting in Alberta is usually from 100 to 300 ft deep. It is completed with blank surface casing, which might be from 50 to 100 ft in length, and is open to the saturated zone over an interval of about 50 to 250 ft. A diagram of a representative well in its hydrogeological setting is shown in figure 3.

CHARACTERISTICS OF PUMP TESTS

Data from pump tests of several hours' duration conducted in the heterogeneous rocks of Alberta exhibit a wide variety of shapes when plotted on a semilogarithmic scale. Four basic shapes were recognized in a study of 122 pump tests in the Edmonton area (Bibby, 1977) and are shown in figure 4. In all of the tests the drawdown readings were taken in the pumped well. These shapes are found for pump tests in heterogeneous strata in all parts of the province. As judged by the amount of drawdown induced and by the slope of the final straight-line portion of the curve, types 1 and 2 appear to be associated with relatively high permeability, and types 3 and 4 with relatively low permeability. Not all drawdown curves fit precisely into this classification, so a degree of subjective judgment is involved.

The recovery curves for all types of drawdown curve invariably have the shape that would be expected if, at the end of pumping, the residual drawdown was computed by extrapolating the last straight-line portion of the drawdown curve and linearly superimposing an imaginary recharge well having exactly the same shape as the drawdown curve. These recovery curves, plotted as residual drawdown versus log (t + t'/t), are shown in figure 4.

The reason for the occurrence of these different types of drawdown curve is not known, but this behavior of the recovery curves is taken to prove that all varieties of drawdown curve reflect conditions in the aquifer and not external factors such as well-bore conditions.

Some of the processes that operate when a well is pumped in these heterogeneous strata can be understood by referring to figure 3. At an approximately horizontal boundary between a sandstone lens and the shale matrix, a process akin to leakage or cross-formational flow takes place. A roughly vertical boundary between a lens and the shale behaves like a partial barrier boundary. In fractured zones turbulent flow can occur. Beds at the water table can be drained. In sandstone lenses intercepted by the well, water is released by compressibility effects at early

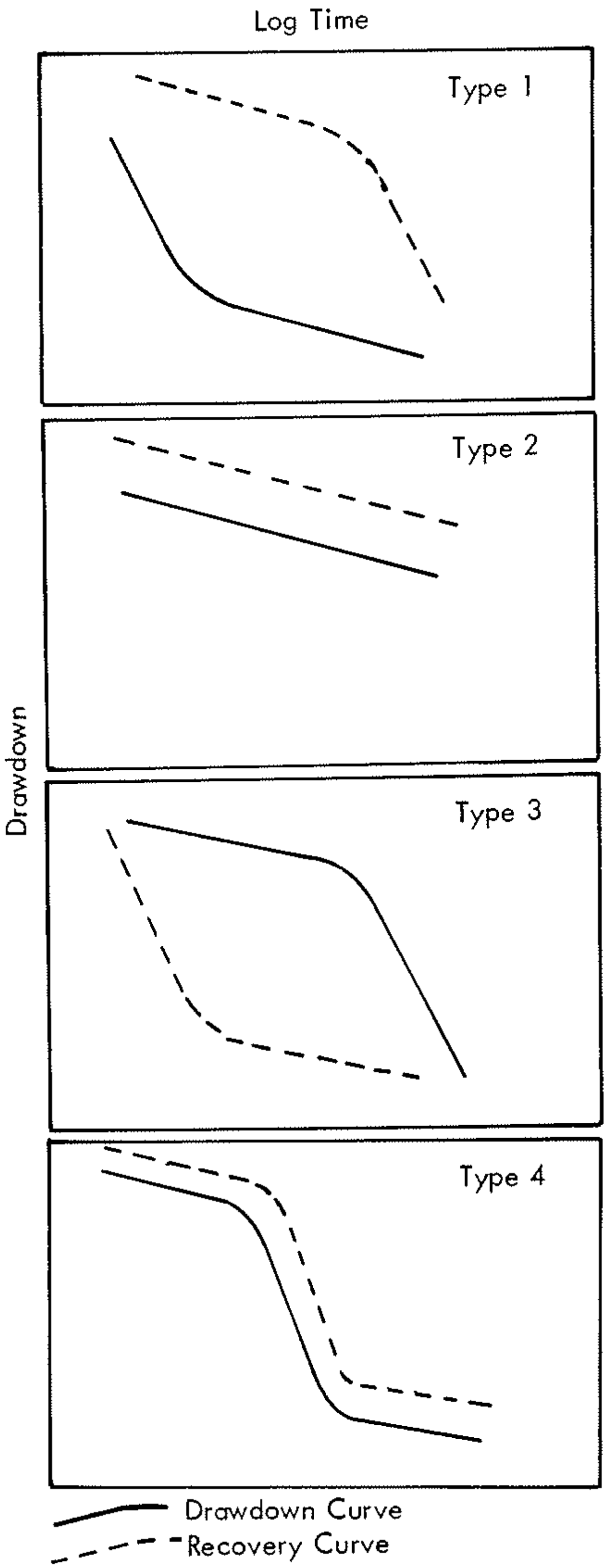


Figure 4. Typical drawdown and recovery curves from pump tests of several hours duration

times in the pump test. All these processes happen in a very complicated time-varying manner which determines the shape of drawdown curve. This hydraulic situation is so highly complex that it cannot be accommodated within any of the models for pump test analysis. The only meaningful parameter that can be used is the drawdown curve itself; it is the signature of the well and is different for every well.

Within a small area the slopes of the final straight-line portions of the drawdown curves have a log-normal frequency distribution. Thus, if these slopes are used in Jacob's equation to compute transmissivity (the meaning of transmissivity in this context is discussed in a following section), then transmissivity likewise has a log-normal distribution.

Information on pump tests of several days' duration is not so common and, naturally, is only available for wells which indicate relatively high permeability in shorter tests. Usually, however, on a semilogarithmic plot the straight-line portion observed in the shorter test continues for some time, eventually deviates from being straight, becomes successively steeper in a transition zone, and finally forms a straight line again (Fig. 5). This final straight line is always steeper than the initial one. Within a small area slopes of the final straight lines from a number of tests tend to converge (Tóth, 1966), and certainly exhibit much less variability than the slopes of the initial straight lines. When plotted on a log-log scale, the portions of the curves corresponding to the straight lines can be fitted by Theis nonleaky type curves.

Drawdowns in observation wells open over approximately the same depth interval as a pumping well give a two-dimensional approximation of the drainage volume caused by pumping. As might be expected, this volume is always highly irregular. Figure 6 gives examples of the development of the drainage volume during pumping.

The variable lithology and the complex hydraulic response to pumping inhibit the use of existing methods to analyze pump tests in the heterogeneous flow situations under consideration, although attempts have been made. In particular, the Theis model and the image-well theory have been used, on the assumption that the observed changes in slope of the plotted drawdown curves indicate the presence of barrier boundaries. However, the integer ratios that should result are invariably not obtained, indicating either that the changes in slope result from causes other than distinct boundaries or that the Theis model is inappropriate.

Usually data are analyzed by fitting either Jacob straight lines or Theis nonleaky type curves to the segments of the plotted drawdown curves. The associated formulas are then used to obtain values of transmissivities and coefficients (for Theis), while acknowledging that the assumptions on which the formulas are based are not met. Used in this way, these methods are strictly

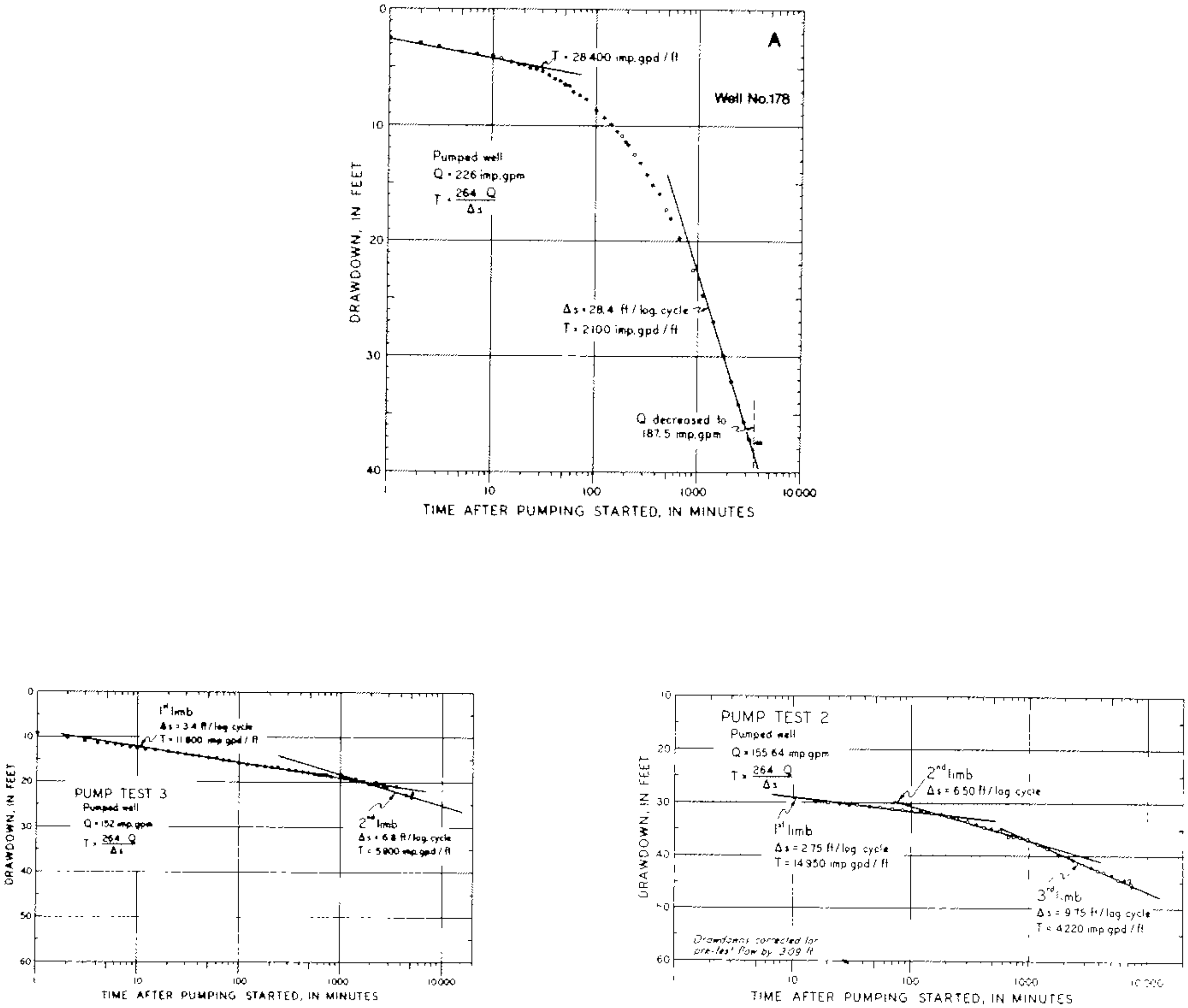


Figure 5. Examples of drawdown curves from pump tests of several hours' duration

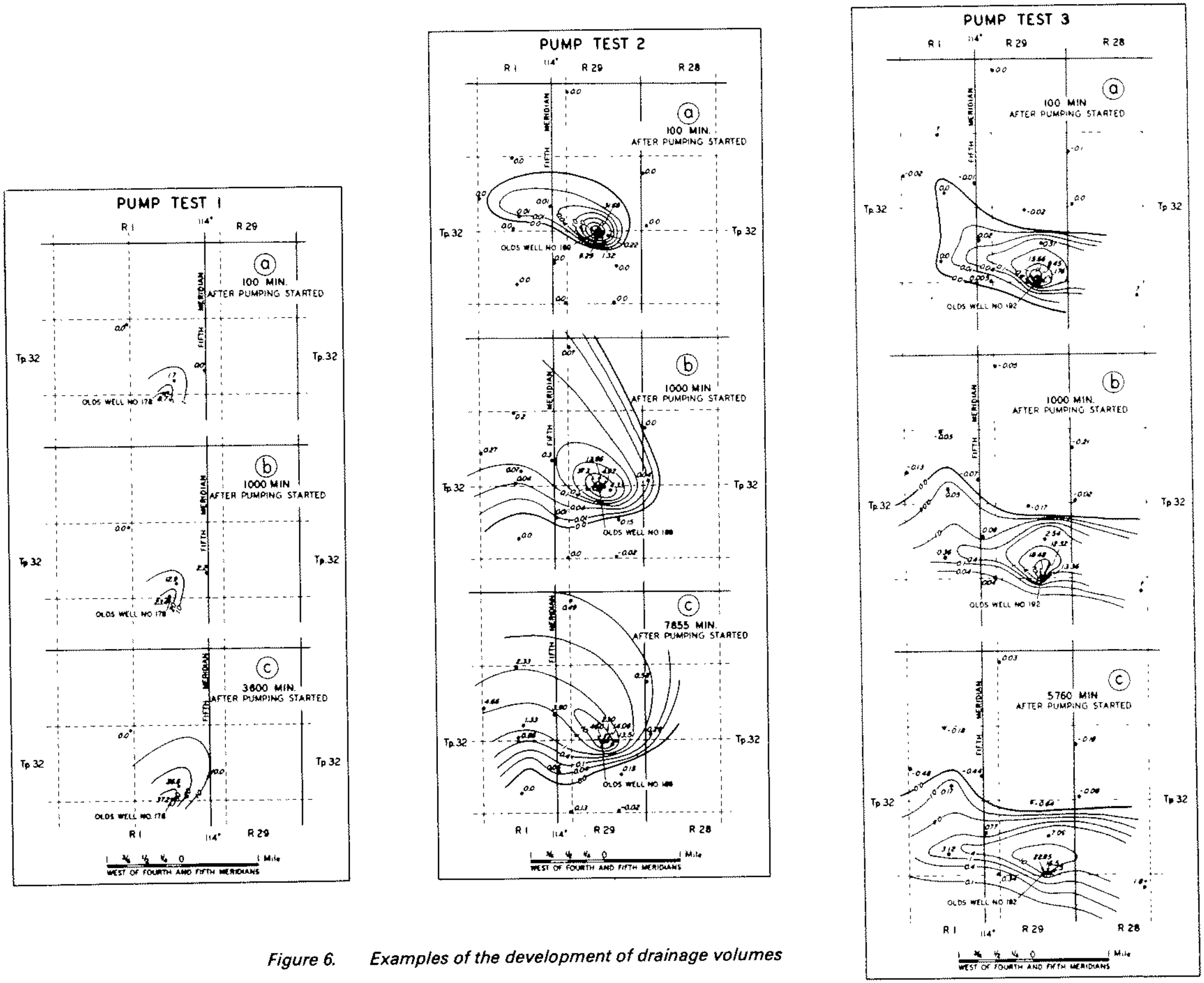


Figure 6. Examples of the development of drainage volumes

formulas, since there is no physical justification for their use. The transmissivity and storage coefficient values that are obtained are simply numbers, having no more meaning than the slope of the drawdown curve from which they are derived.

An example of the application of Jacob's method to a typical drawdown curve is shown in figure 7 where, for simplicity's sake, a pump test rate of 100 igpm is assumed. The graph shows the initial straight-line section running from 7 min to 250 min, the transition zone running from 250 min to 700 min, and the final straight-line section running from 700 min to 10,080 min. As shown on the graph, a typical two-hour test produces only the initial straight-line section and gives no indication of the slope of the final straight-line section. A typical seven-day test produces the initial straight-line section, the transition zone, and the final straight-line section.

Jacob's formula for transmissivity is (Ferris *et al.*, 1962):

$$T = \frac{2.30Q}{4\pi ds} \times \log_{10} \frac{(t_2)}{(t_1)}$$

for consistent units, and

$$T \approx \frac{264Q}{\Delta s}$$

for field units, where

T = transmissivity (igpd/ft)

Q = pump test rate (igpm)

Δs = drawdown per log cycle in min (ft)

ds = drawdown from time t_1 to time t_2 (length)

From the graph the slopes of the first and second straight-line sections are 0.5 ft/log cycle and 5 ft/log cycle respectively. Thus, Jacob's formula gives a transmissivity of 52,800 igpd/ft based on the initial straight line and 5280 igpd/ft based on the final straight line. The first transmissivity value probably reflects the average or effective transmissivity of the rock mass near the well and is referred to as the *local* transmissivity. With this interpretation, the values of transmissivity obtained from all tests of a few hours' duration may be considered to be *local* transmissivities. The transmissivity value obtained from the final straight line is interpreted as indicating the

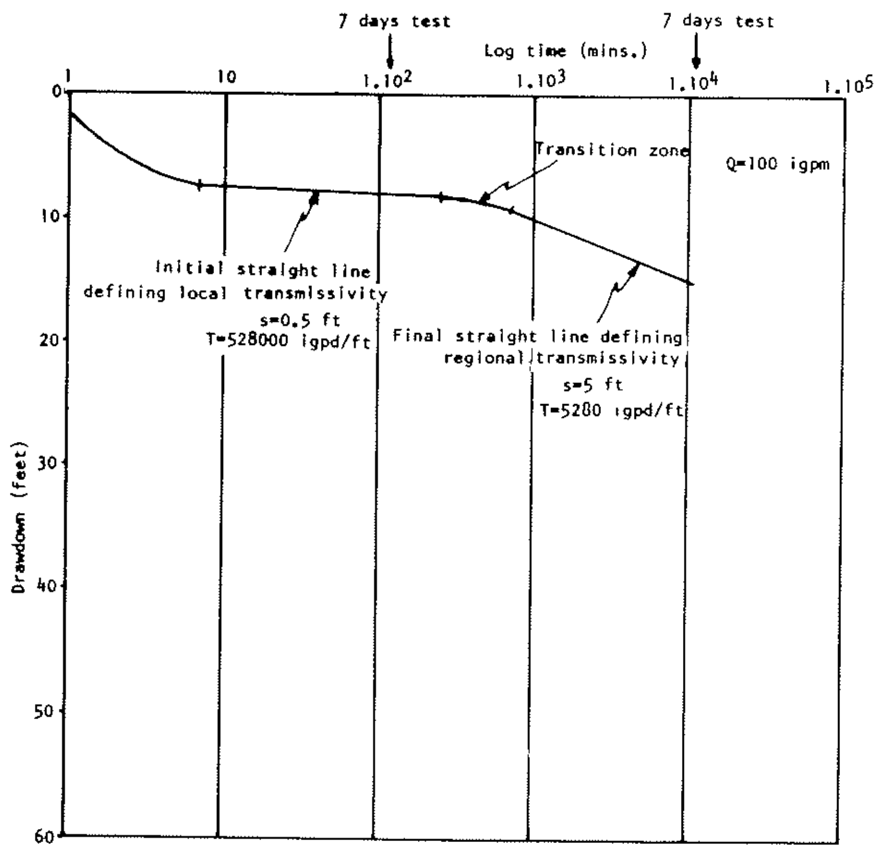


Figure 7. The use of Jacob's formula to determine local and regional transmissivity

average or effective transmissivity of the large volume of rock being influenced at these later times. This calculated transmissivity is referred to as the *regional* transmissivity. The transition limb is attributed to the drainage volume reaching a zone of facies change.

The method of predicting sustainable pumping rates is illustrated in figure 8. It is assumed that the slope of the final straight line, and therefore the regional transmissivity, will remain unchanged from the end of the pump test of seven days (approximately 4 log-cycles) to 20 years (approximately 7 log-cycles). The principle that the drawdown at a specific time is linearly related to the pumping rate is then applied. This principle enables the 20-year sustainable pumping rate to be calculated using the formula,

$$Q_{20} = Q \frac{A_d}{S_{20,Q}}$$

where,

Q_{20} = 20-year sustainable pumping rate (igpm)

A_d = available drawdown (ft)

$S_{20,Q}$ = extrapolated drawdown at 20 years at the pump rate, Q.

In practice it is not necessary to draw the graph to 1×10^7 min, because

$$S_{20,Q} = S_{1000} + 4\Delta s$$

where,

S_{1000} = drawdown in the pump test at 1000 min (ft)

Δs = slope of the final straight line (ft)

so that,

$$Q_{20} = Q \frac{A_d}{S_{1000} + 4\Delta s}$$

For the example, S_{1000} = 10 ft, Δs = 5 ft/log cycle and $S_{20,Q}$ = 30 ft, so that Q_{20} = 200 igpm.

Figure 8 shows that the expected drawdown curve for a pumping rate of 200 igpm can be obtained simply by shifting the pump test drawdown curve and its extrapolated portion so that the curve passes through S = 60 ft (the available drawdown) at t = 1×10^7min.

The 20-year period is used almost universally in Alberta for calculating sustainable yields. The selection and

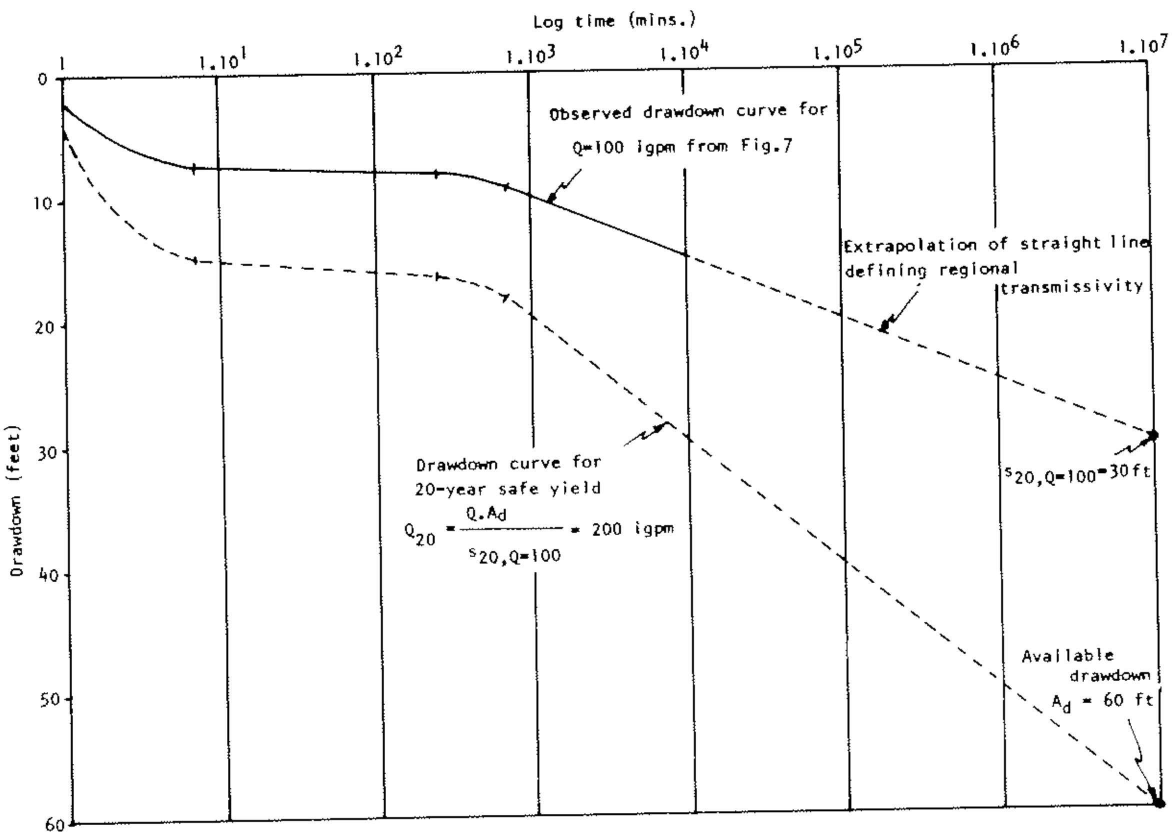


Figure 8. The currently used method of calculating a 20-year safe yield

continued use of this value is certainly arbitrary, but figure 8 shows that choosing a longer period, say 30 years, makes little difference on the semilog scale, and a very much shorter value would be considered too short to be called sustainable. The 20-year period has the advantages of being a round number and of being very nearly equal to seven log-cycles.

Predictions of 20-year sustainable yields apply only to the well on which the test was conducted and cannot be applied to other points in the neighborhood because, even if the regional transmissivity value were the same, local transmissivity values are too variable (having a log-normal distribution). The highly irregular cone of depression also prevents the prediction of drawdown at points in the aquifer away from the pumping well.

The weakness of this method of predicting 20-year sustainable yields is its implicit extrapolation of the final straight-line section of the drawdown curve over the remaining part of the 20-year period. Data obtained from water level and production records over a period of years indicate that further steepening of the drawdown curve takes place. Nevertheless, the drawdown curve over the last few log-cycles will likely approach a straight line because the drainage volume at these times is so large that its average conductive capability approaches a constant.

Significantly, the method of predicting 20-year sustainable yields, as distinct from that used to calculate local and regional transmissivities, does not depend on Jacob's equation. The sole assumption made is that if the drawdown curve is presumed known for 20 years for any constant pumping rate, then the 20-year sustainable yield can be obtained by a *linear* shift of this curve. For the specific purpose of predicting Q_{20}, the calculation of transmissivity is a redundant step. This again indicates that the drawdown curve is the essence of well hydraulics in highly heterogeneous systems.

Using the formula developed by Theis to analyze data from observation wells gives exactly the same values of local and regional transmissive capacity as using Jacob's formula. Values of storage coefficient calculated by the formula vary greatly with time and the distance from the pumping well. Tóth (1966) reported values ranging from 0.0001 to 0.5 for a single pump test. This large variance shows that the Theis method is inappropriate for this situation.

TERMINOLOGY

The description of the hydrogeologic situation in Alberta for the purpose of evaluating well yield in heterogeneous sedimentary rocks shows that current hydrogeologic terminology is inadequate and that misunderstanding is caused by persisting with its use. Terms such as "aquifer", "transmissivity", "leakage", "barrier boundary", "confined", "unconfined", and "water table" are all to some extent inappropriate. The two basic reasons for this are: (1) the definitions of many of these terms are based partly or wholly on simple geologic criteria which are not applicable to these complex heterogeneous strata and (2) the scale of observation is crucial when studying heterogeneous systems because system behavior changes with volume or duration. For example, the strata are highly heterogeneous on a small local scale, but may appear to be homogeneous on the large scale of the regional flow system. Or, the response to pumping can be dominated in the early stages by compressibility effects, which is a characteristic of truly confined aquifers, but by drainage effects later, which is a characteristic of water-table aquifers.

In the context of this study, the terms that are most misleading are "aquifer" and "transmissivity". The term "aquifer", defined as "a geological unit containing and capable of transmitting economic amounts of water", is not valid because no such unit is identifiable using geological criteria. The "unit" which contains and transmits groundwater is the entire saturated zone which is bounded not by a geological boundary but by the water table. In the absence of a better term, "aquifer" is used in this report with the understanding that it has no geological connotation and refers to the entire saturated zone.

"Transmissivity", because it is a property of an aquifer, also has in this context no meaning. The term is, however, generally used in Alberta, and so has been used thus far in this report in describing the Alberta hydrogeologic setting. However, the term has been qualified by the adjectives "local" and "regional", which shows that the problem of definition has been recognized but not yet solved. The convention of using the term appears to have grown out of the common use of the Jacob and Theis equations.

To avoid confusion, it is proposed that in Alberta the term "transmissive capacity" be used to refer to the concept that is now called "transmissivity". As used here, "transmissive capacity" refers to the average capability of a rock mass to conduct water. No restriction is placed on the size or shape of the rock mass or on the nature of the averaging and no specification of its detailed conductive capability is required. In this report this term usually refers to a single well, and indicates the average conductive capability of the drainage volume caused by pumping the well. The value is, in general, the number obtained from Jacob's formula. Transmissive capacity thus varies with the duration of pumping, so is a function of time. To qualify the term relative to time, the adjectives short-term, medium-term, and long-term are used. A short-term pump test has a duration of hours, a medium-term test of days, and a long-term test of years (production records). The terms, therefore, differ by orders of magnitude in the time scale and correspond to the durations of pumping tests normally encountered. With these qualifiers, a short-term pump test gives rise to a short-term drainage volume, allowing the calculation of a short-term transmissive capacity, and similarly for medium-term and long-term pump tests. With these definitions, local transmissivity becomes short-term transmissive capacity and regional transmissivity becomes medium-term transmissive capacity.

STATEMENT OF THE PROBLEM

The problem, simply put, is to devise a means of predicting sustainable pumping rates to wells in the heterogeneous, clastic strata in Alberta. No theoretical methods exist, and the empirical method currently used involves a straight-line extrapolation which is known to be invalid.

In view of the overall hydrogeological setting and the type of information obtainable in such a setting, the problem can be divided into two components:

1) to develop a method or methods for estimating the later portion of the drawdown curve from some easily observable parameter; and
2) to develop a method of predicting drawdown behavior for any arbitrary, long-term production program.

The later portion of the drawdown curve cannot be measured directly because of the large periods of time that would be required so must be estimated. Once obtained, this estimate can be used to calculate a single value of long-term sustainable pumping rate, which is useful for assessing the resources of an area. In practice, however, production wells are operated at variable rates even over the long term, so any predictive methods, to be of value, must take these variable rates into account.

ESTIMATING LONG-TERM TRANSMISSIVE CAPACITY

DEVELOPMENT OF THE METHOD

The first consideration is to select the parameter to be used to estimate the transmissive capacity of the long-term drainage volume. From the previous description of the situation in Alberta, the obvious choice is the short-term transmissive capacity. Values of this parameter are available in reasonable quantity and are relatively easy to obtain. In addition, because this parameter is a function of the hydraulic characteristics of the aquifer in the same physical way, except for scale, as the long-term transmissive capacity, it is reasonable to expect that a predictive relationship exists between the two.

In order to develop this predictive relationship, it is necessary to define the concept of equivalent transmissive capacity and to measure the average of the set of data.

The equivalent transmissive capacity of a drainage volume is the transmissive capacity of a hypothetical homogeneous drainage volume that has the same dimensions as the real heterogeneous drainage volume and that will pass the same flux under the same pressure drop (Cardwell and Parsons, 1945; Warren, Skiba and Price, 1961). It has also been called the 'effective' (Warren and Price, 1961) and 'integral' (Toronyi and Farouq Ali, 1974) transmissive capacity. Where permeability or some other hydraulic parameter is used instead of transmissive capacity, the analogous term is equivalent permeability.

The equivalent transmissive capacity is therefore an average transmissive capacity of the drainage volume under specified boundary conditions. However, the precise method of averaging is not specified. For example, consider the following three explicit measures of the average of a set of transmissive capacity values, τ_i, $i = 1 \ldots t$:

$$\text{Arithmetic Average} = A(\tau) = \frac{\sum_{i=1}^{t} w_i \tau_i}{\sum_{i=1}^{t} w_i}$$

$$\text{Geometric Average} = G(\tau) = \left(\prod_{i=1}^{t} \tau_i^{\,w_i} \right)^{1/\sum_{i=1}^{t} w_i}$$

$$\text{Harmonic Average} = H(\tau) = \frac{\sum_{i=1}^{t} w_i}{\sum_{i=1}^{t} w_i/\tau_i}$$

where, w_i $i = 1 \ldots t$ are weights.

These averages are such that,

$$A(\tau) \geq G(\tau) \geq H(\tau)$$

For homogeneous layers, the arithmetic average is the equivalent transmissive capacity of the layers arranged in parallel while the harmonic average is the equivalent transmissive capacity of the layers arranged in series. This is shown diagrammatically in figure 9 for four homogeneous blocks, two having transmissive capacity τ_1 and two transmissive capacity τ_2. For the general case of a porous medium having any number of different block transmissive capacities arranged randomly and any type of directional variation of flow, Cardwell and Parsons (1945) showed that the equivalent transmissive capacity lies between the harmonic and arithmetic averages of the actual transmissive capacities. For homogeneous blocks of different size, the averages have to be weighted for volume and, in the case of radial flow, for distance. Cardwell and Parsons further state that because simple mathematical considerations do not favor either the upper or lower bound, the equivalent transmissive capacity must be estimated.

Warren and Price (1961) showed that the permeability obtained from pressure buildup analysis is a reasonable measure of the equivalent permeability of the drainage volume. This was demonstrated using three-dimensional numerical models composed of homogeneous blocks of differing permeabilities. The permeabilities were generated randomly from selected probability density functions. The model of a given spatial distribution of permeabilities was then used to calculate both equivalent and pressure buildup permeabilities.

Warren and Price (1961) also confirmed with the same models that equivalent permeability depends in general on the spatial arrangement of permeabilities: for every rearrangement of the homogeneous blocks a different equivalent permeability was obtained. The only exception to this was for a homogeneously heterogeneous drainage volume, which consists of such a large number of small homogeneous blocks that any rearrangement of the blocks does not change the equivalent permeability. They concluded, however, that for any situation or variety of probability density functions, the best estimate of the expected value of equivalent permeability was the geometric mean of the permeabilities of the blocks composing the drainage volume. In addition they found that geometry, anisotropy, and partial penetration had a finite but not significant influence on the expected value of equivalent permeability. In the extreme case of zero vertical permeability and 30 percent partial penetration, the expected value of equivalent permeability was decreased by less than 40 percent. In more normal circumstances much smaller decreases were noted.

Taken together, these two findings indicate that the best estimate of pressure buildup permeability is the geometric mean of the permeabilities of the blocks of the drainage volume. This conclusion is of course subject to the assumptions made: namely, that the drainage volume can be adequately modeled by a collection of homogeneous blocks and that the lateral and vertical permeability variations are the same order of magnitude. The first assumption requires only that enough blocks be included in the model. The second assumption depends upon the specific situation but is certainly valid in many circumstances.

These conclusions of Warren and Price are adaptable to the groundwater situation in Alberta. The adaptation consists primarily of modifying the assumptions leading to the development of the geometric mean as a random variable whose parameters, and therefore the expected value of long-term transmissive capacity, are estimated from a random sample of short-term transmissive capacities. Applications to field cases are used to evaluate the method.

It is assumed that the long-term drainage volume is composed of n homogeneous blocks of equal size, having transmissive capacities, T_i, $i = 1 \ldots n$. T_1 is considered to be the transmissive capacity of the center block, containing the well at which the estimation of long-term transmissive capacity is to be made, and is considered to be known. The T_i s of the blocks are considered to be unknown but to have a log-normal frequency distribution. This assumption about the nature of the frequency distribution is made in order to allow the development of the properties of the geometric mean as a random variable. The choice of the log-normal distribution is based on the observed frequency distribution of short-term transmissive capacities at a number of sites in Alberta.

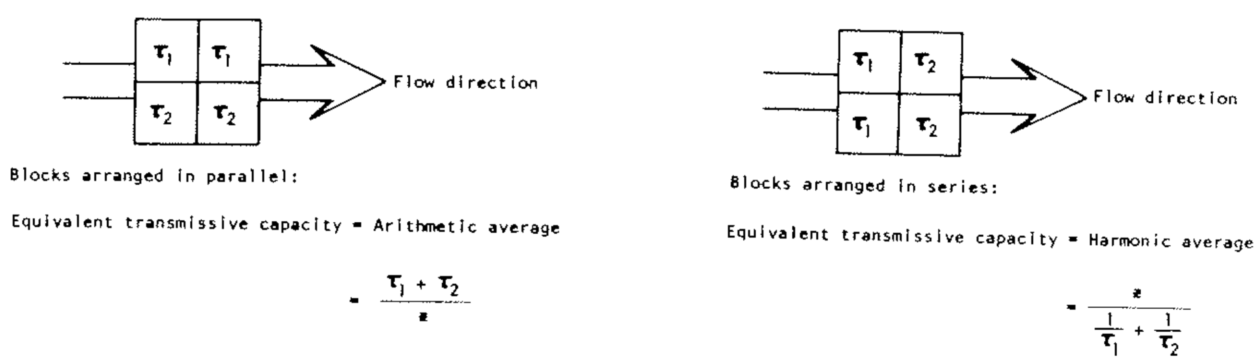


Figure 9. Equivalent transmissive capacities of two arrangements of homogeneous blocks

ARITHMETIC DEVELOPMENT

If Y is a random variable with a normal probability density function, $f_y(Y)$, a mean μ_y, and variance σ_y^2, that is

$$Y \sim N(\mu_y, \sigma_y^2),$$

$$f_y(Y) = \frac{1}{\sqrt{2\pi}\,\sigma_y} \exp\{-\frac{1}{2\sigma_y^2}(Y-\mu_y)^2\},$$

then, if $Y = \log_e X$, X has a log-normal distribution, $f_x(X)$, that is,

$$X \sim \Lambda(\mu_y, \sigma_y^2)$$

$$f_x(X) = \frac{1}{\sqrt{2\pi}\,x\sigma_y} \exp\{-\frac{1}{2\sigma_y^2}(\log_e X - \mu_y)^2\}$$

$$E(X) = \alpha_x = \exp\{\mu_y + \tfrac{1}{2}\sigma_y^2\}$$

$$Var(X) = \beta_x^2 = \alpha_x^2(\exp(\sigma_y^2) - 1).$$

A log-normal distribution with $\mu_y = \sigma_y^2 = 0.5$ is shown in figure 10. Aitchison and Brown (1957) state the following theorem which is of use. If X_j, $j = 1 \ldots n$ are independent random variables with a log-normal distribution, that is,

$$X_j \sim \Lambda(\mu_{yj}, \sigma_{yj}^2), \; j = 1 \ldots n,$$

and,

$$Z = c \prod_{j=1}^{n} X_j^{b_j},$$

where,

$c = e^a$ is a constant

b_j, $j = 1 \ldots n$ are constants

and

$$\sum_{j=1}^{n} b_j \mu_{yj} \text{ and } \sum_{j=1}^{n} b_j^2 \sigma_{yj}^2 \text{ converge,}$$

then,

$$Z \sim \Lambda\left[a + \sum_{j=1}^{n} b_j \mu_{yj}, \sum_{j=1}^{n} b_j^2 \sigma_{yj}^2\right]$$

Note that,

$$\alpha_Z = \exp\{a + \sum_{j=1}^{n} b_j \mu_{yj} + \tfrac{1}{2}\sum_{j=1}^{n} b_j^2 \sigma_{yj}^2\}$$

$$\beta_Z = \alpha_Z(\exp\{\sum_{j=1}^{n} b_j^2 \sigma_{yj}^2\} - 1)^{1/2}.$$

In the event that the values of all the T_i are known, but not their locations, the best estimate of the expected value of long-term transmissive capacity is the unweighted geometric mean,

$$G = \left[\prod_{i=1}^{n} T_i\right]^{1/n}$$

This is the result of Warren and Price. Since only T_1 is assumed known and the remainder to have a log-normal frequency distribution, the geometric mean has to be modified in two ways: first, to introduce weights to allow for the fact that the value and location of T_1 are known and second, to treat T_j, $j = 2 \ldots n$ as random outcomes from a log-normal frequency distribution. This results in the geometric mean becoming:

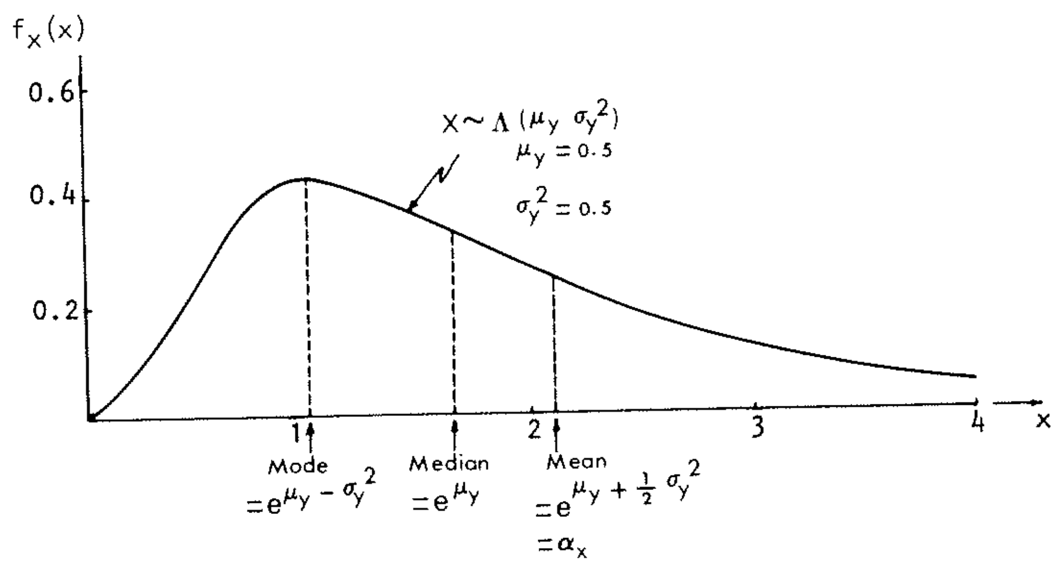


Figure 10. A log-normal frequency curve

$$G = T_1^{\,w_1/\sum_{i=1}^{n} w_i} \left[\prod_{j=2}^{n} T_j^{w_j} \right]^{1/\sum_{i=1}^{n} w_i}$$

where w_i, $i = 1 \ldots n$, are weights which are as yet unspecified.

Letting the log-normal frequency distribution be:

$$T \sim \Lambda(\mu_Y, \sigma_Y^2)$$

where,

$$Y = \log_e T \sim N(\mu_Y, \sigma_Y^2)$$

then, by the foregoing theorem, G has a log-normal distribution,

$$G \sim \Lambda(\mu_F, \sigma_F^2)$$

where,

$$F = \log_e G \sim N(\mu_F, \sigma_F^2)$$

$$\mu_F = C_1 \log_e T_1 + C_2 \mu_Y$$

$$\sigma_F^2 = 2C_3 \sigma_Y^2$$

$$E(G) = \alpha_G = \exp\{C_1 \log_e T_1 + C_2 \mu_Y + C_3 \sigma_Y^2\}$$

$$\mathrm{Var}(G) = \beta_G^2 = \exp\{2C_1 \log_e T_1 + 2C_2 \mu_Y + 4C_3 \sigma_Y^2\}$$

$$- \exp\{2C_1 \log_e T_1 + 2C_2 \mu_Y + 2C_3 \sigma_Y^2\},$$

where,

$$C_1 = \frac{w_1}{\sum_{i=1}^{n} w_i}$$

$$C_2 = \frac{\sum_{j=2}^{n} w_j}{\sum_{i=1}^{n} w_i} = 1 - C_1$$

$$C_3 = \frac{\sum_{j=2}^{n} w_j^2}{2 \cdot \left[\sum_{i=1}^{n} w_i \right]^2}$$

It is now necessary to develop estimators of the parameters of G. This is done by assuming that the short-term transmissive capacity values τ_i, $i = 1 \ldots t$, $t << n$, are equivalent to a random sample of the homogeneous blocks of the drainage volume. That is, τ_i, $i = 1 \ldots t$, are a sample from $T \sim \Lambda(\mu_Y, \sigma_Y^2)$, with $T_1 = \tau_1$. Then, maximum likelihood estimates of μ_Y and σ_Y^2 are (Aitchison and Brown, 1957):

$$\hat{\mu}_Y = \frac{1}{t} \sum_{i=1}^{t} \log \tau_i$$

$$\hat{\sigma}_Y^2 = \frac{1}{t-1} \sum_{i=1}^{t} (\log \tau_i - \hat{\mu}_Y)^2 .$$

From the invariance property of maximum likelihood estimators it follows that:

$$\hat{\mu}_F = C_1 \log T_1 + C_2 \hat{\mu}_Y$$

$$\hat{\sigma}_F^2 = 2 C_3 \hat{\sigma}_Y^2.$$

Using the maximum likelihood method to develop estimators for β_G^2 and α_G leads to difficulties, but Finney (1941) has outlined an equivalent method. It makes use of the fact that $\hat{\mu}_Y$ and $\hat{\sigma}_Y^2$ are jointly sufficient and that any function of such estimators is the minimum variance unbiased estimator of its expectation. Thus, if $h(\hat{\mu}_Y, \hat{\sigma}_Y^2)$ is some function of $\hat{\mu}_Y, \hat{\sigma}_Y^2$ and $E(h(\hat{\mu}_Y, \hat{\sigma}_Y^2)) = \Theta$, then $h(\hat{\mu}_Y, \hat{\sigma}_Y^2)$ is the minimum variance unbiased estimate of Θ. It is therefore necessary to find a function with expectation α_G and one with expectation β_G^2. It is shown in Appendix 1 that such functions are:

$$\hat{\alpha}_G = \exp\{C_1 \log T_1 + C_2 \hat{\mu}_Y\} \, \Psi_t\left(\hat{\sigma}_Y^2 \left\{ \frac{tC_3 - C_2^2}{t} \right\} \right)$$

$$\hat{\beta}_G^2 = \exp\{2C_1 \log T_1 + 2C_2 \hat{\mu}_Y\} \left[\Psi_t\left(\left\{ \frac{4tC_3 - 4C_2^2}{t} \right\} \hat{\sigma}_Y^2 \right) - \Psi_t\left(\left\{ \frac{2tC_3 - 4C_2^2}{t} \right\} \hat{\sigma}_Y^2 \right) \right]$$

where,

$$\Psi_t(S) = 1 + S + \frac{1}{2!} \frac{(t-1)}{(t+1)} S^2 + \frac{1}{3!} \frac{(t-1)^2}{(t+1)(t+3)} S^3 + \ldots\ldots$$

An exact confidence interval can be obtained for μ_F in the following way:

since,

$$\hat{\mu}_Y \sim N\left(\mu_Y, \frac{\sigma_Y^2}{t}\right)$$

then,

$$\hat{\mu}_F = C_1 \log T_1 + C_2 \hat{\mu}_Y \sim N\left(\mu_F, \frac{C_2^2 \sigma_Y^2}{t}\right)$$

since,

$$(t-1)\cdot\frac{\hat{\sigma}_Y^2}{\sigma_Y^2} \sim X^2\,(t-1)$$

then,

$$\frac{\dfrac{\hat{\mu}_F - \mu_F}{C_2\sigma_Y/\sqrt{t}}}{\sqrt{\dfrac{\hat{\sigma}_Y^2}{\sigma_Y^2}}} \sim t_{t-1}$$

so that an exact percent confidence interval on μ_F is:

$$\frac{\hat{\mu}_F - t_{p,t-1}\, C_2\, \hat{\sigma}_Y}{\sqrt{t}}, \quad \frac{\hat{\mu}_F + t_{p,t-1} \cdot C_2 \cdot \hat{\sigma}_Y}{\sqrt{t}}$$

where $t_{p,t-1}$ is the appropriate percentage point from a t-distribution with (t−1) degrees of freedom.

An exact confidence interval cannot be obtained for α_G. For large samples an approximate interval can be obtained (see Aitchison and Brown, 1957).

The specification of these estimators allows the expected value of long-term transmissive capacity to be estimated. In general it is unjustifiable even to suggest that the arithmetic mean (α_G in this case) is the best estimate of a quantity that can be derived from a set of measurements with a distribution that is not normal (Gaddum, 1945). When a logarithmic transformation leads to normality, it follows that the median (exp (μ_F) in this case) rather than the mean is the most likely value. For this reason, the best estimate of the expected value of long-term transmissive capacity is exp $\{\hat{\mu}_F\}$. However, for the field cases examined, the other estimators are also evaluated.

It is now necessary to define the weights w_i. According to Cardwell and Parsons (1945), the transmissive capacity of each homogeneous block should be weighted directly in proportion to block volume and inversely by the square of the radial distance to the center block. In practice there is no basis for defining different block sizes so it has been assumed that all blocks have equal volume. To define radial distance, some kind of geometry for the blocks must be specified. There are any number of choices. The one that has been used is simple and physically realistic: the drainage volume is divided into m concentric rings centered on the well at which the estimate of long-term transmissive capacity is to be made (fig.11).

The radius of each ring is:

$$ia,\ i = 1 \ldots m,$$

where, a is a constant.

The area of each ring is:

$$(2\,i - 1)\,\pi a^2,\ i = 1 \ldots m.$$

The number of blocks of equal size per ring is:

$$(2\,i - 1),\ i = 1 \ldots m.$$

The total number of blocks in the model is:

$$n = \sum_{i=1}^{m} (2\,i - 1) = m^2.$$

The mean radial distance to each block in a ring is r_i, where

$$r_i^2 = \frac{a^2}{2}\left(i^2 + (i-1)^2\right),\ i = 1 \ldots m$$

so that,

$$w_k = \frac{1}{r_i^2} = \frac{2}{a^2\,(i^2 + (i-1)^2)},$$

$$k = \{1 + \sum_{p=1}^{i-1} (2p-1)\}, \{\sum_{p=1}^{i} (2p-1)\};\ i=1 \ldots n$$

Then,

$$\sum_{k=1}^{n} w_k = \sum_{i=1}^{m} (2i-1)\,\frac{2}{a^2\,(i^2 + (i-1)^2)}$$

and

$$C_1 = \frac{1}{\displaystyle\sum_{i=1}^{m} \frac{(2i-1)}{i^2 + (i-1)^2}},$$

$$C_2 = \frac{\displaystyle\sum_{p=2}^{m} \frac{(2p-1)}{p^2 + (p-1)^2}}{\displaystyle\sum_{i=1}^{m} \frac{2i-1}{i^2 + (i-1)^2}} = 1 - C_1,$$

and

$$C_3 = \frac{\displaystyle\sum_{p=2}^{m} \frac{(2p-1)^2}{(p^2 + (p-1)^2)^2}}{2 \cdot \left[\displaystyle\sum_{i=1}^{m} \frac{2i-1}{i^2 + (i-1)^2}\right]^2}$$

Values of C_1, C_2, and C_3 for various values of m are given in table 1. With the aid of this table, the estimate of the expected value of long-term transmissive capacity, exp ($\hat{\mu}_F$), can easily be calculated from a sample of short-term transmissive capacities.

This definition of the weights requires the specification of 'a' and r_{dv} = a.m, the outer radius of the drainage volume. Once r_{dv} is fixed, the choice of 'a' determines the degree of heterogeneity since it fixes the number of blocks, n. In practice, for realistic choices of r_{dv} and 'a' (say, $r_{dv} \simeq 10$ miles, $a \simeq 1$ mile, m = 10), the values of C_1, C_2, and C_3 are not very sensitive (Table 1), so that the values of the estimators are likewise not very sensitive. The choice is thus not critical.

Table 1. C_1, C_2, and C_3 for Values of m

m	C_1	C_2	C_3
1	1.0000	0.0	0.0
2	0.6250	0.3750	0.070313
3	0.5039	0.4961	0.064479
4	0.4416	0.5584	0.057164
5	0.4026	0.5974	0.051412
6	0.3753	0.6247	0.046979
7	0.3549	0.6451	0.043490
8	0.3390	0.6610	0.040677
9	0.3260	0.6740	0.038357
10	0.3152	0.6848	0.036408
11	0.3061	0.6939	0.034744
12	0.2981	0.7019	0.033304
13	0.2912	0.7088	0.032043
14	0.2851	0.7149	0.030928
15	0.2796	0.7204	0.029934
16	0.2746	0.7254	0.029040
17	0.2701	0.7299	0.028232
18	0.2660	0.7340	0.027496
19	0.2623	0.7377	0.026823
20	0.2588	0.7412	0.026204
30	0.2342	0.7658	0.021922
40	0.2194	0.7806	0.019443
50	0.2092	0.7908	0.017780
60	0.2015	0.7985	0.016565
70	0.1954	0.8046	0.015628
80	0.1905	0.8095	0.014875
90	0.1863	0.8137	0.014254
100	0.1827	0.8173	0.013729
200	0.1622	0.8378	0.010881
300	0.1522	0.8478	0.009599
400	0.1458	0.8542	0.008819
500	0.1412	0.8588	0.008277
600	0.1377	0.8623	0.007869
700	0.1348	0.8652	0.007547
800	0.1324	0.8676	0.007284
900	0.1304	0.8696	0.007063
1000	0.1286	0.8714	0.006874

Table 2. Limiting Values of Parameters of $f_g(g)$

Parameter	n = 1	n → ∞
C_1	1	0
C_2	0	1
C_3	0	0
μ_F	log T_1	μ_Y
$\exp(\mu_F)$	T_1	$\exp(\mu_Y)$
σ_F^2	0	0
α_G	T_1	$\exp(\mu_Y)$
β_G^2	0	0

It should be noted that this definition of weights does not imply that flow is quasi-radial in a plane; it is merely a mechanism for fixing w_i, i=1 . . . n. However, the converse is true. For quasi-radial flow in a plane, the w_i as defined would be correct.

In certain circumstances it might be more reasonable to define the weights by inventing a different model, for example, layered or spherical or by assuming knowledge of more block transmissive capacities. This would change w_i and n but not the general form of G.

It is interesting to note the behavior of the parameters of $f_G(g)$ in the limit when n = 1 and when n → ∞ (Table 2). For n = 1 the drainage volume is homogeneous, being composed of a single block. For n → ∞ the volume is homogeneously heterogeneous. At both limits $f_G(g)$ becomes a spike: for n = 1, G = T, and for n → ∞, G = $\exp(\mu_Y)$, the median of $f_T(t)$.

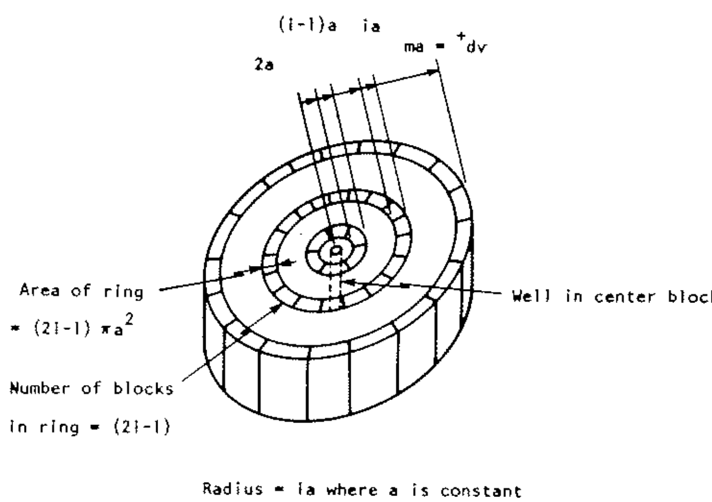


Figure 11. Concentric ring model of a drainage volume

So far attention has been focussed on the geometric mean since this is the estimate of the *expected* value of long-term transmissive capacity. The work of Cardwell and Parsons (1945) indicated that upper and lower bounds on *possible* values of long-term transmissive capacity are the arithmetic and harmonic means. (Note here the distinction between the "expected value" and "possible values" of long-term transmissive capacity).

Under the same assumptions as those made to derive the geometric mean, the arithmetic mean takes the form:

$$A = T_1 \frac{w_1}{\sum_{i=1}^{n} w_i} + \frac{\sum_{j=2}^{n} T_j w_j}{\sum_{i=1}^{n} w_i}$$

The frequency distribution of A does not take a simple form, so only its mean and variance are derived.

$$E\,(A) = T_1 \frac{w_1}{\sum_{i=1}^{n} w_i} + \alpha_T \frac{\sum_{j=2}^{n} w_j}{\sum_{i=1}^{n} w_i}$$

where,

$$\alpha_T = \exp\{\mu_Y + \tfrac{1}{2}\sigma_Y^2\}\,.$$

Aitchison and Brown (1957) give the equivalent maximum likelihood estimator of α_T as:

$$\hat{\alpha}_T = \exp\{\hat{\mu}_Y\} \cdot \phi_t(\tfrac{1}{2}\hat{\sigma}_Y^2)$$

so that,

$$\hat{E}(A) = T_1 C_1 + \hat{\alpha}_T \cdot C_2$$

and

$$\text{Var}\,(A) = \frac{\sum_{j=2}^{n} w_j^2}{\left(\sum_{i=1}^{n} w_i\right)^2} \cdot \beta_T^2\,,$$

where,

$$\beta_T^2 = \exp\{2\mu_Y + \sigma_Y^2\} \cdot \left(\exp(\sigma_Y^2) - 1\right).$$

Aitchison and Brown (1957) give the equivalent maximum likelihood estimator of β_T^2 as:

$$\hat{\beta}_T^2 = \exp\{2\hat{\mu}_Y\}\left(\phi_t(2\hat{\sigma}_Y^2) - \phi_t\left(\frac{n-2}{n-1}\hat{\sigma}_Y^2\right)\right)$$

so that,

$$\widehat{\text{Var}}(A) = \frac{\sum_{j=2}^{n} w_j^2}{\left(\sum_{i=1}^{n} w_i\right)^2} \cdot \hat{\beta}_T^2$$

$$\phi_t(S) = 1 + \frac{t-1}{t} \cdot S + \frac{(t-1)^3}{t^2 \cdot (t+1)} \frac{S^2}{2!} + \frac{(t-1)^5}{t^3(t-1)\,(t+3)} \frac{S^3}{3!} + \ldots\ldots$$

The harmonic mean has the form:

$$H = \frac{\sum_{i=1}^{n} w_i}{\frac{w_1}{T_1} + \sum_{j=2}^{n} \frac{w_j}{T_j}}$$

The frequency distribution of H does not take a simple form and unfortunately, its mean and variance cannot be derived in closed form. For this reason the only way to estimate E(H) and Var(H) is by the Monte Carlo method. This involves repeatedly generating sets of random values of T_j, $j = 2 \ldots n$ from $f_T(t;\, \hat{\mu}_Y, \hat{\sigma}_Y^2)$ and calculating the corresponding values of H until a random sample of H is obtained. From this sample E(H) and Var(H) are estimated by the sample mean and variance. For the form of H given here, generating the sample and the estimates is very simple, but, due to the multiplicity of calculations, requires the use of a computer.

The various estimates of long-term transmissive capacity can now be used to calculate corresponding values of sustainable pumping rates.

If it is assumed that a time-drawdown curve is available on the well located in the center block (from either a short-term or medium-term pumping test) and that the available drawdown is given (Fig. 12), then the 20-year safe yield can be calculated from:

$$Q_{20} = \frac{A_d}{\frac{S_t}{Q} + \frac{\Delta s_L}{Q}\,(7 - \log t)}$$

where,

$$\Delta s_L = \frac{264Q}{T_L}$$

T_L = long-term transmissive capacity (igpd/ft)

Q = pump test rate (igpm)

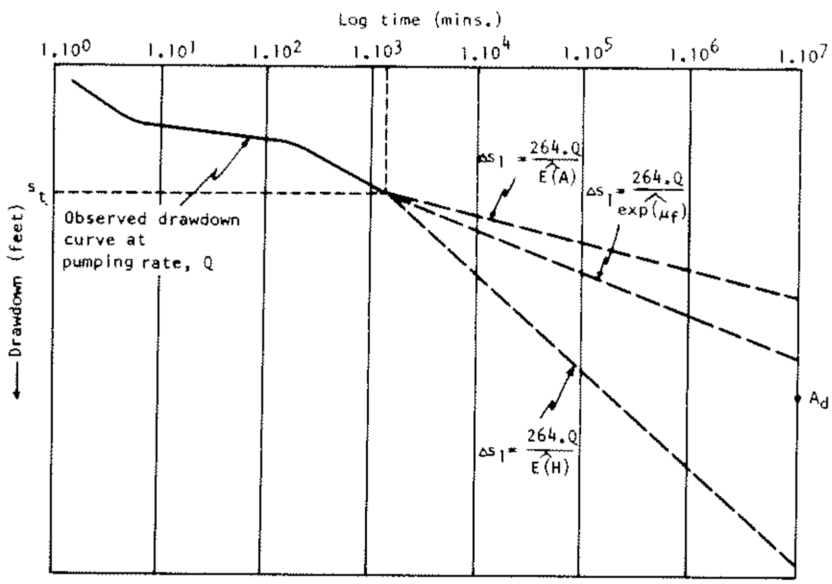
A_d = available drawdown (ft)

t = duration of pump test (min)

S_t = drawdown at time t (ft)

Q_{20} = 20-year safe yield (igpm)

Δs_L = slope per log cycle of drawdown curve corresponding to long-term transmissive capacity (ft).



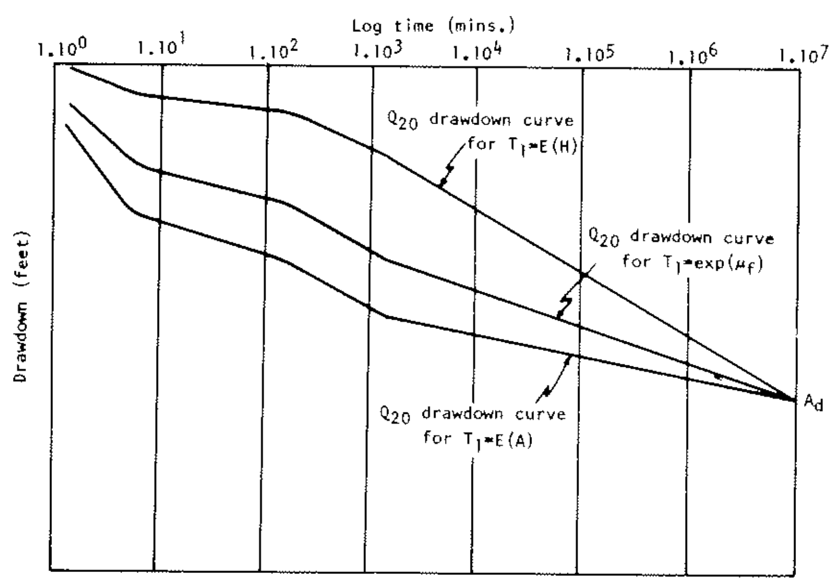


Figure 12. Calculation of Q_{20} using estimates of long-term transmissive capacity

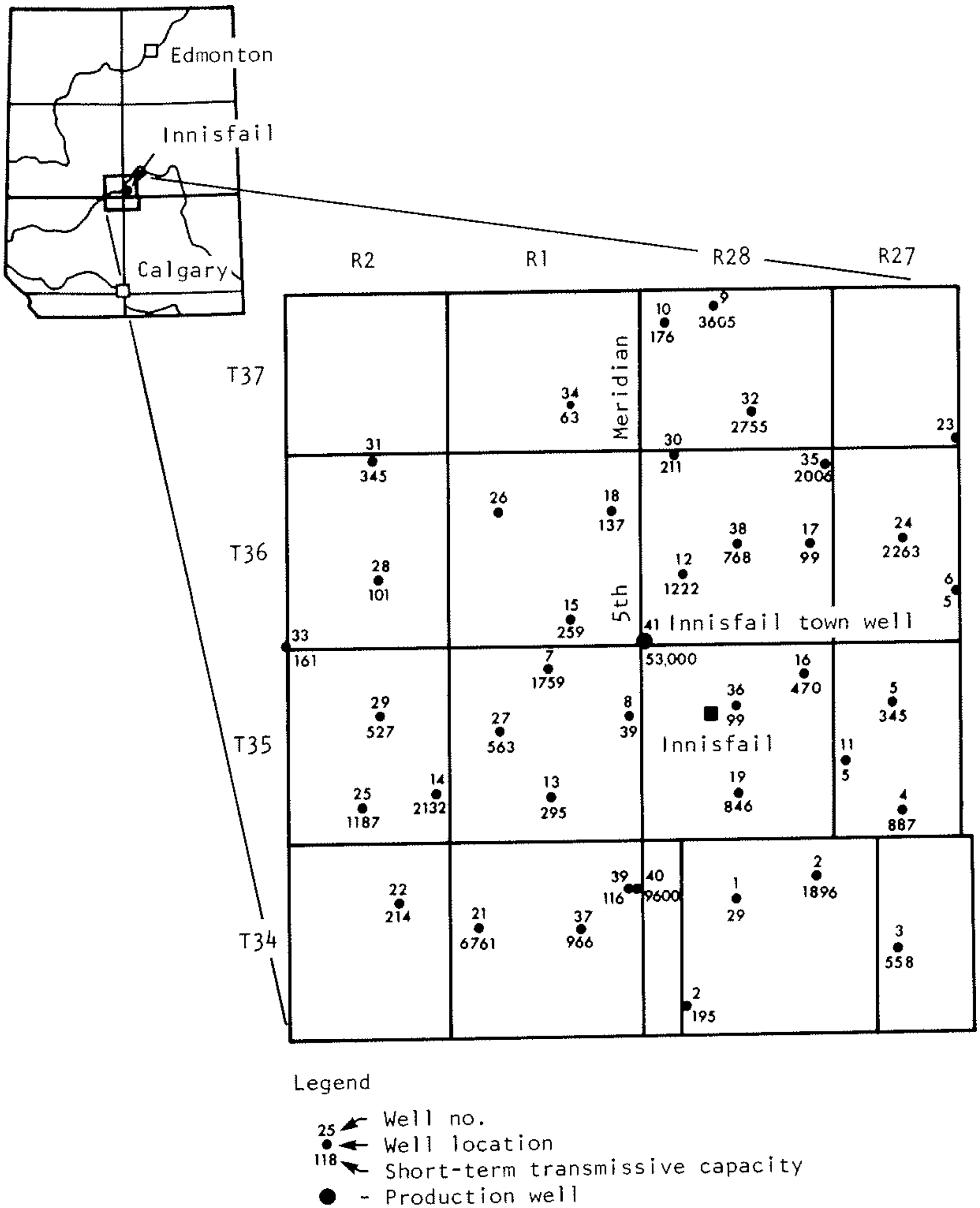


Figure 13. Innisfail: well locations and short-term transmissive capacities

By making appropriate substitutions for T_L, one can obtain the estimate of sustainable yield corresponding to the estimate of expected value of long-term transmissive capacity, exp $(\hat{\mu}_F)$, the estimates of upper and lower bounds on long-term transmissive capacity $(\widehat{E(A)}, \widehat{E(H)})$ and so on.

A summary of the steps involved in applying the method is given in appendix 2.

FIELD EVALUATION OF THE PROPOSED METHOD

The only way to check the validity of this method of predicting long-term transmissive capacity is to examine field cases, but finding examples in Alberta suitable for the purpose is very difficult. To be a good field case for verification purposes, a production well must have both long-term production records and long-term water level records as well as a random sample of short-term transmissive capacity values. The production and water level records should be of several years' duration so that the long-term transmissive capacity can be calculated and compared to the estimate based on short-term transmissive capacity values. Even with complete records production rates will likely have been variable, so the calculation of transmissive capacity is not a formal procedure. Normally, however, records are incomplete or are complicated by factors such as interference.

Short-term transmissive capacity values are usually available near production wells having long records but typically not in sufficient quantities to form a good random sample. However, one reason for choosing short-term transmissive capacity as the parameter with which to estimate long-term transmissive capacity was that it is readily observable.

There are in fact only two sites in Alberta for which the available information comes close to satisfying these requirements: Olds and Innisfail. Both of these will be described and estimates developed. However, only at Innisfail are the hydrogeological conditions suited to the method; at Olds an hydraulic barrier cuts across the aquifer so the method is unsuitable for this case because it cannot account for such discontinuities. Nevertheless the Olds situation will be described in order to demonstrate an implicit premise of the method: the flow region must be fully hydraulically connected.

Field Case Number 1: Innisfail

Innisfail, a town of 2500 in central Alberta, is 70 mi north of Calgary (Fig. 13) in Tp 35, R 28, W 4th Mer. Since 1965 a single well, located about 3 mi northwest of the town on the banks of the Red Deer River, has supplied the town with water.

The water source well is completed in the bedrock which is the Paskapoo Formation of Tertiary age. This formation is composed of bentonitic sandstones, claystones, and siltstones and is continental in origin.

The well is 145 ft deep with blank casing from land surface to 90 ft and an open hole from 90 to 145 ft. The producing horizon is reported by the driller to be 40 ft of sandstone.

Also shown in figure 13 are the locations of wells for which short-term transmissive capacities are available. Most of these transmissive capacity values were obtained in the fall of 1974 in a pump test program conducted by the Alberta Research Council, which accounts for the availability of a large number of values with a uniform areal distribution, and also demonstrates the relative ease with which the parameter can be measured. There is, on average, one value every 8 sq mi within a 10-mi radius of the production well.

The short-term transmissive capacity values range in magnitude from 5 to 53,000 igpd/ft. The sample was tested for log-normality by the chi-square test and found to have a log-normal probability distribution. The pertinent information on the test is given in table 3. The time-drawdown data of the pump tests, from which the short-term transmissive capacities were calculated, are shown in appendix 3. The production and water level records for the Innisfail town well are the best in the entire province. Records are complete over the life of the well. Water levels have been measured continuously by

Table 3. Innisfail: Chi-square Test of Short-term Transmissive Capacity Data

Logarithms Of Short-Term Transmissive Capacities											
Class Mid-Points											
1.5	2.375	3.25	4.125	5.0	5.875	6.75	7.625	8.5	9.375	10.25	11.125
Frequency											
2	1	2	1	11	6	8	6	2	1	0	1

Number of observations = 41

X^2 = 7.7526

Number of degrees of freedom = 6

Probability = .25679

automatic recorder on an observation well located 15 ft from the production well. The record starts in August, 1965, about 60 days after production commenced. Production has been measured daily on a volumetric basis.

Figure 14 is a graph of the water level and production records. The water levels are the values at the end of every month taken during a pumping cycle. The spikes on the plot correspond to periods when the pump was not operating. Note that water levels have continuously but gradually declined so that at the present time the level is about 40 ft below the original static level. The plot of production data is the average of the daily volumes in every month. The zero values on the plot are for periods when the pump was not operating. Note also that production has gradually increased. Over the life of the well the production has averaged about 22,864 ft^3/day (99 igpm). In late 1965 the rate was about 17,400 ft^3/day (76 igpm); in summer 1973 it was about 31,000 ft^3/day (135 igpm). To the end of 1973 the well produced about 73 million ft^3 of water. The increase in the volume of production over the life of the well can be approximated by a linear function as shown in figure 14.

For the analysis of the long-term record the water levels are shown on a semilog plot in figure 15. If the increase in the rate of production is kept in mind, the data can be fitted very well by a straight line from 70 days ($t = 1 \times 10^5$ min) to the end of the record. The slope of this straight line, Δs, is

$$\Delta = 16.25 \text{ ft/log-cycle.}$$

The average production rate is $Q = 22{,}864$ ft^3/day (99 igpm) so that the long-term transmissive capacity is:

$$T = \frac{264(99)}{16.25} = 1608 \text{ igpd/ft}$$

$$\simeq 1600 \text{ igpd/ft.}$$

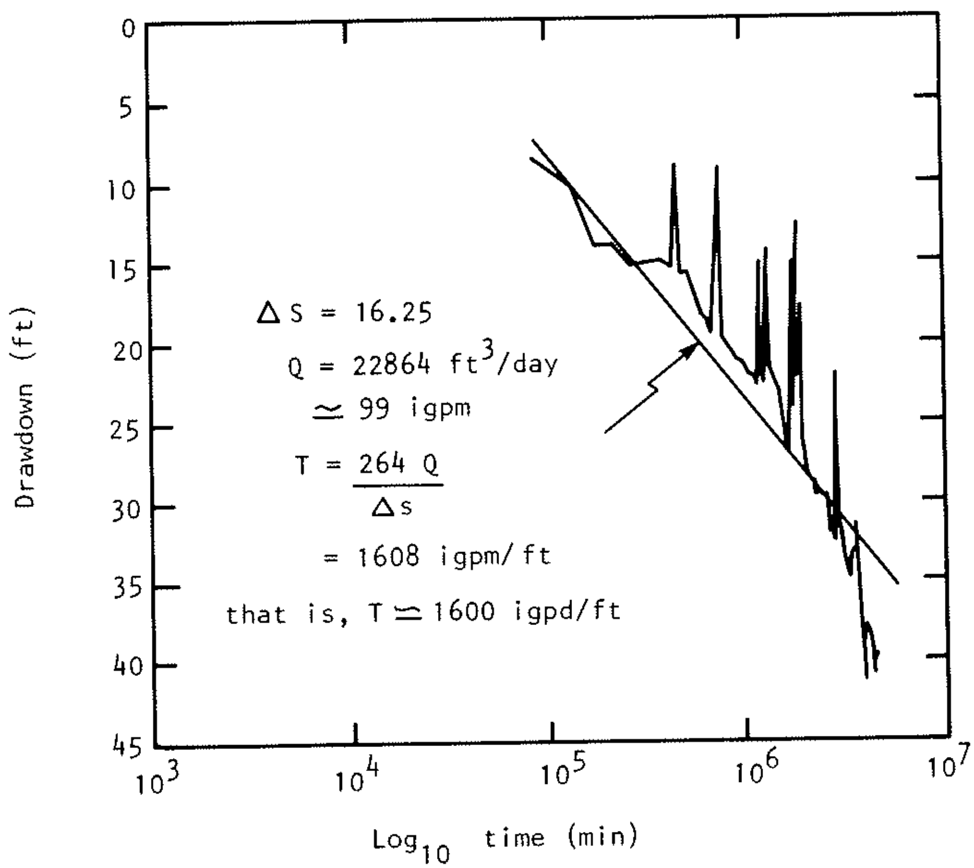


Figure 15. Innisfail: semilog plot of drawdowns

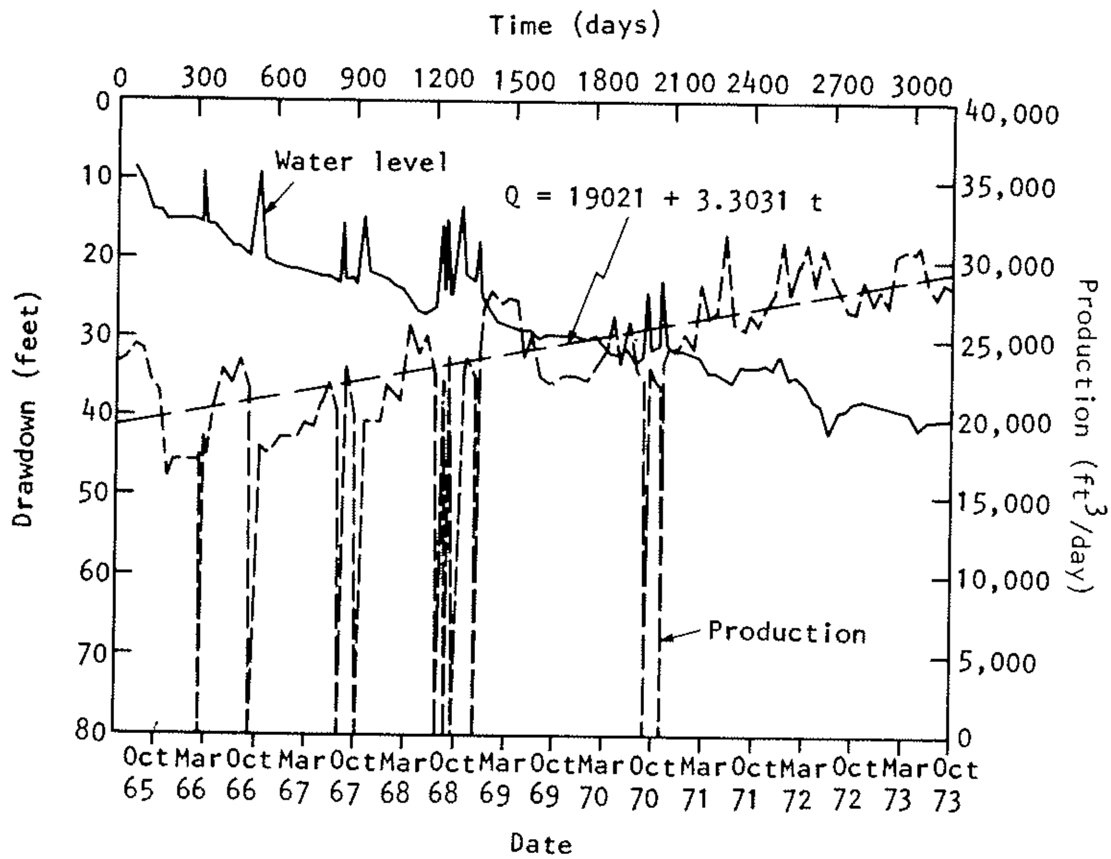


Figure 14. Innisfail: production and water level records

Estimates of the parameters of G, A, and H based on the sample of 41 short-term transmissive capacities are given in table 4 and figure 16 for various values of m, the number of rings of the model.

All the estimates show a monotonic decrease as m increases, but, except for the first few values of m, the estimates do not change very much over fairly large ranges of m. This insensitivity of the estimators to m is very significant, since the selection of a and r_{dv}, and therefore m, is somewhat arbitrary.

The estimator of the expected value of long-term transmissive capacity, $\exp(\hat{\mu}_F)$, is closest to the observed value (T_L = 1600 igpd/ft) for m = 13 ($\exp(\hat{\mu}_F)$ = 1596 igpd/ft). For r_{dv} = 10 mi, this would give a ring size of a = 0.77 mi, which is quite reasonable. The change in $\exp(\hat{\mu}_F)$ with increasing m is so small that all values of m from 9 to 20 give estimates within about 15 percent of the observed long-term transmissive capacity. The 95 percent confidence interval on $\exp(\hat{\mu}_F)$ for m = 13 is 1034 to 2464 igpd/ft.

The estimate of the mean of G, $\hat{\alpha}_G$, is very close to the estimate of the median, $\exp(\hat{\mu}_F)$, over the whole range of m, so that it would make little difference if the mean instead of the median of G had been used to estimate the expected value of long-term transmissive capacity. The closeness of the estimates results from the fact that $\hat{\sigma}_F$ is very small relative to $\hat{\mu}_F$.

For each m the estimates of the parameters of H were obtained by the Monte Carlo method from a sample of size 50. For this reason there is some noise about the overall monotonic decrease of the estimates. Due to limitations on available computer time and costs, the estimates were computed only up to m = 20. The estimate of the lower bound of possible values of long-term transmissive capacity for m = 13 is $\widehat{E(H)}$ = 107 igpd/ft. The estimate of the upper bound is $\widehat{E(A)}$ = 17,003 igpd/ft. This very wide range results from the fact that the value of T_1 (53,000 igpd/ft) is very large compared to the median of the frequency distribution of T ($\exp(\hat{\mu}_y)$ = 378 igpd/ft), and characteristically the arithmetic mean is influenced predominantly by large values and the harmonic mean by small ones.

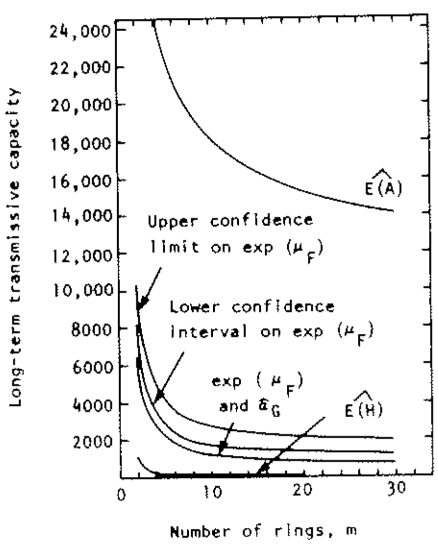


Figure 16. Innisfail: estimates of geometric, arithmetic and harmonic means

The values of 20-year safe yield corresponding to the various estimates of long-term transmissive capacity are shown in table 5. For these calculations it was assumed known that at 60 days after pumping commenced ($t = 1 \times 10^5$ min) the drawdown was 8 ft. These values are the first observation of the long-term water level record (Fig. 14). The average pumping rate over this 60-day period was 76 igpm. The available drawdown for the well was taken to be 80 ft, the difference between the static water level prior to production and the top of the open interval of the well.

Table 5 shows that the Q_{20} values decrease monotonically with increasing m and are not very sensitive to changes in m. For m = 13 the estimate of Q_{20} corresponding to $\exp(\hat{\mu}_F)$ is 183 igpm and is of course more or less the same as the Q_{20} based on $\hat{\alpha}_G$. The 95 percent confidence interval for this value of m is 130 to 250 igpm. The lower and upper bounds on possible values of Q_{20} are 16 and 587 igpm.

Application of the method to the Innisfail situation reveals that it gives excellent estimates of the expected value of long-term transmissive capacity and therefore the expected value of 20-year safe yield. The Innisfail case is a severe test of the method because short-term transmissive capacities range over four orders of magnitude and the value of T_1 is very much greater than $\exp(\hat{\mu}_y)$. The latter factor causes a situation in which a short-term transmissive capacity of 53,000 igpd/ft decreases to a long-term value of 1600 igpd/ft. Such a divergence is unlikely to be approached, let alone exceeded, in most situations, so the method can be applied with confidence.

The Innisfail case also reveals that the method will not always give a good *practical* estimate of the limits of possible long-term transmissive capacity values. The estimates of upper and lower bounds of possible values of long-term transmissive capacity are seen to be significantly influenced by a large divergence between T_1 and $\exp(\hat{\mu}_y)$ which does not invalidate them but gives a much wider range to possible values of long-term transmissive capacity than would occur with all but

Table 4. Innisfail: Estimates of the Parameters of Geometric, Arithmetic, and Harmonic Means

Number of rings	Number of nodes	$\widehat{E(H)}$ [1]	Lower confidence interval on $\exp(\mu_F)$	$\widehat{\exp(\mu_F)}$ [2]	Upper confidence interval on $\exp(\mu_F)$	$\hat{\alpha}_G$ [3]	$\widehat{E(A)}$ [4]	$\widehat{Var(H)}$ [5]	$\hat{\sigma}_F$ [6]	$\hat{\beta}_G$ [7]	$\sqrt{\widehat{Var(A)}}$ [8]
1	1	-	-	53000	-	53000	-	-	-	-	-
2	4	1043	6601	8306	10453	8956	33955	2160	.42	3970	4000
3	9	543	3368	4565	6187	4794	27804	281	.38	1830	1800
4	16	253	2383	3355	4725	3459	24640	179	.34	1190	1640
5	25	158	1918	2767	3990	2817	22658	81	.32	891	1520
6	36	136	1649	2418	3546	2440	21275	73	.30	722	1420
7	49	144	1472	2187	3247	2192	20240	68	.29	614	1360
8	64	129	1347	2021	3030	2015	19429	71	.27	539	1300
9	81	120	1254	1895	2865	1882	18771	59	.26	484	1250
10	100	117	1181	1797	2734	1778	18223	59	.25	442	1210
11	121	121	1122	1717	2628	1694	17758	39	.25	409	1180
12	144	106	1074	1651	2539	1625	17355	47	.24	382	1150
13	169	107	1034	1596	2464	1567	17003	45	.24	359	1120
14	196	116	999	1548	2400	1517	16691	33	.23	340	1100
15	225	106	969	1507	2343	1474	16413	32	.23	324	1080
16	256	97	943	1470	2294	1436	16161	43	.22	310	1060
17	289	97	919	1438	2249	1403	15933	36	.22	298	1040
18	324	91	899	1409	2210	1373	15725	35	.22	287	1030
19	361	96	880	1383	2174	1346	15533	44	.21	277	1010
20	400	94	863	1360	2141	1322	15357	38	.21	268	1000
30	900	-	753	1204	1925	1162	14109	-	.19	213	907
40	1600	-	694	1119	1806	1075	13359	-	.18	184	850
50	2500	-	655	1064	1728	1019	12839	-	.17	166	811
60	3600	-	628	1024	1671	979	12448	-	.16	153	781
70	4900	-	607	994	1628	949	12140	-	.16	144	757
80	6400	-	591	970	1593	924	11888	-	.15	136	738
90	8100	-	577	950	1565	904	11675	-	.15	130	722
100	10000	-	565	934	1541	888	11493	-	.15	125	706
200	40000	-	505	843	1410	798	10450	-	.13	98	628
300	90000	-	477	803	1350	757	9942	-	.12	89	590
400	160000	-	461	778	1313	732	9618	-	.12	82	565
500	250000	-	449	760	1287	715	9385	-	.11	78	547
∞	-	-	-	378	-	378	-	-	-	-	-

All values of estimators are in igpd/ft

T_I = 53000 igpd/ft

$\hat{\mu}_Y$ = 5.94

$\hat{\sigma}_Y$ = 1.94

Sample size = 41

$t_{40,.95}$ = 2.02

[1] Estimate of lower bound on possible values of long-term transmissive capacity

[2] Estimate of expected value of long-term transmissive capacity

[3] Estimate of mean of G

[4] Estimate of upper bound on possible values of long-term transmissive capacity

[5] Estimate of standard deviation of H

[6] Estimate of standard deviation of F

[7] Estimate of standard deviation of G

[8] Estimate of standard deviation of A

Table 5. Innisfail: Values of 20-year Safe Yields Based on Estimates of Long-term Transmissive Capacity

Number of rings	$\widehat{E(H)}$ [1]	Lower confidence interval on $\exp(\mu_F)$	$\exp(\widehat{\mu_F})$ [2]	Upper confidence interval on $\exp(\mu_F)$	$\hat{\alpha}_G$ [3]	$\widehat{E(A)}$ [4]
1	-	-	-	-	-	-
2	131	432	474	514	487	662
3	49	305	362	420	371	644
4	34	245	305	369	310	631
5	23	210	270	337	273	622
6	20	188	247	315	249	615
7	21	172	231	299	231	609
8	19	161	218	286	218	604
9	18	152	208	276	207	600
10	17	145	200	268	199	596
11	18	139	194	261	192	593
12	16	134	188	255	186	590
13	16	130	183	250	181	587
14	17	126	179	246	176	584
15	16	123	176	242	173	582
16	14	120	172	238	169	580
17	14	118	169	235	166	578
18	14	115	167	232	163	576
19	14	113	164	230	161	574
20	14	112	162	227	158	573
30	-	99	147	211	143	561
40	-	92	139	201	134	553
50	-	88	133	195	128	546
60	-	85	129	190	124	542
70	-	82	126	186	121	538
80	-	80	123	183	118	534
90	-	78	121	181	116	532
100	-	77	119	179	114	529
200	-	69	109	167	104	514
300	-	66	105	161	100	505
400	-	64	102	158	97	500
500	-	62	100	155	95	495

Available Drawdown = 80 ft
$t = 1.10^5$ mins
S_t = 8 ft
Q = 76 igpm

$$Q_{20} = \frac{Ad}{\frac{S_t}{Q} + \frac{264}{T_L}(7 - \log t)} \text{ igpm}$$

[1] Estimate of lower bound on possible values of long-term transmissive capacity
[2] Estimate of expected value of long-term transmissive capacity
[3] Estimate of mean of G
[4] Estimate of upper bound on possible values of long-term transmissive capacity

negligibly small probability. What is really needed, of course, is quantiles on long-term transmissive capacity, but since no direct sample is available these cannot be obtained.

In the practical situation of being required to predict the 20-year safe yield at Innisfail prior to any long-term production, one would have evaluated the estimators for m = 15 to give an estimate of the expected value of Q_{20} of 176 igpm with a 95 percent confidence interval of 123 to 242 igpm and with lower and upper bounds of 16 and 582 igpm. These bounds would be recognized as being much wider than could be expected. The selection of m is arbitrary. The value of m = 15 is chosen because it gives reasonable values of r_{dv} and a and is in a range in which the estimators are not very sensitive to m.

Field Case Number 2: Olds

Olds is a town of about 3500 located 55 mi north of Calgary (Fig. 17) and 18 mi south of Innisfail. The town itself is situated on a north-south ridge at an elevation of 3425 ft and the well field which supplies the town is 2½ mi to the southeast on the eastern flank of the ridge at an elevation of 3200 to 2350 ft.

Wells are completed in the bedrock which is, as at Innisfail, the continental Paskapoo Formation of Tertiary age. The main lithologic components are claystone, bentonitic fine-grained sandstone, and siltstone.

Since 1965 the town has been supplied from three production wells. These were installed following a study by Tóth (1966) in 1964 to 1965. A further study was made in 1970 to 1971 (Tóth, 1973). Because of these studies, a relatively large number of short-term pump and bail tests and long-term water level and production records are available.

The locations of wells at which information pertinent to this study is available are shown in figure 17. In this figure each well is numbered using the same system as Tóth (1966, 1973). The three production wells are numbers 178, 189, and 192. In addition to the well number, the short-term transmissive capacity values are given. An hydraulic barrier is also shown in figure 17 running approximately east-west across the well field and separating well 192 from the other two production wells. The presence and extent of this barrier were determined by Tóth from the response of observation wells during pump tests. He found that wells located on the opposite side of the barrier to a pumped well showed no drawdown.

One of the basic premises of this proposed method of estimating long-term transmissive capacity is complete hydraulic connection within the aquifer. Because the aquifer at Olds does not comply with this premise, this method is inappropriate for the Olds situation. However, the data from Olds can be analyzed to give an estimate of long-term transmissive capacity assuming that no barrier boundary exists. Comparison of this estimate with that obtained from analysis of production and drawdown measurement data on the production wells will give an indication of the significance of this barrier boundary and, consequently, of the limitations on applicability of the method.

The short-term transmissive capacity values shown in figure 17 are obtained from short-term bail and pump tests. The semilog plots of the time-drawdown data for these tests are given in appendix 4. The values of short-term transmissive capacity vary from 27 to 58,667 igpd/ft. This is about the same as at Innisfail but $\hat{\mu}_y$ is greater (8.10 to 5.94) and $\hat{\sigma}_y$ smaller (1.61 to 1.94). The values have a log-normal distribution. The results of the chi-square test for log-normality are given in table 6.

This very wide range in short-term transmissive capacity values is a further confirmation of the high degree of heterogeneity of the aquifer. Figure 17 shows that most of the short-term transmissive capacity values are fairly evenly spaced in a 4 sq mi area near the three production wells. Five of the values to the north are somewhat remote from the production wells, so they are probably not part of the drainage volume of the production wells; however, they are assumed to be from the same statistical population.

Table 6. Olds: Chi-square Test of Short-term Transmissive Capacity Data

Logarithms Of Short-Term Transmissive Capacities							
Class Mid-Points							
3.2	4.27	5.34	6.41	7.48	8.55	9.62	10.69
Frequency							
1	0	1	5	11	4	8	3

Number of observations = 33

χ^2 = 4.5080

Number of degrees of freedom = 3

Probability = .20286

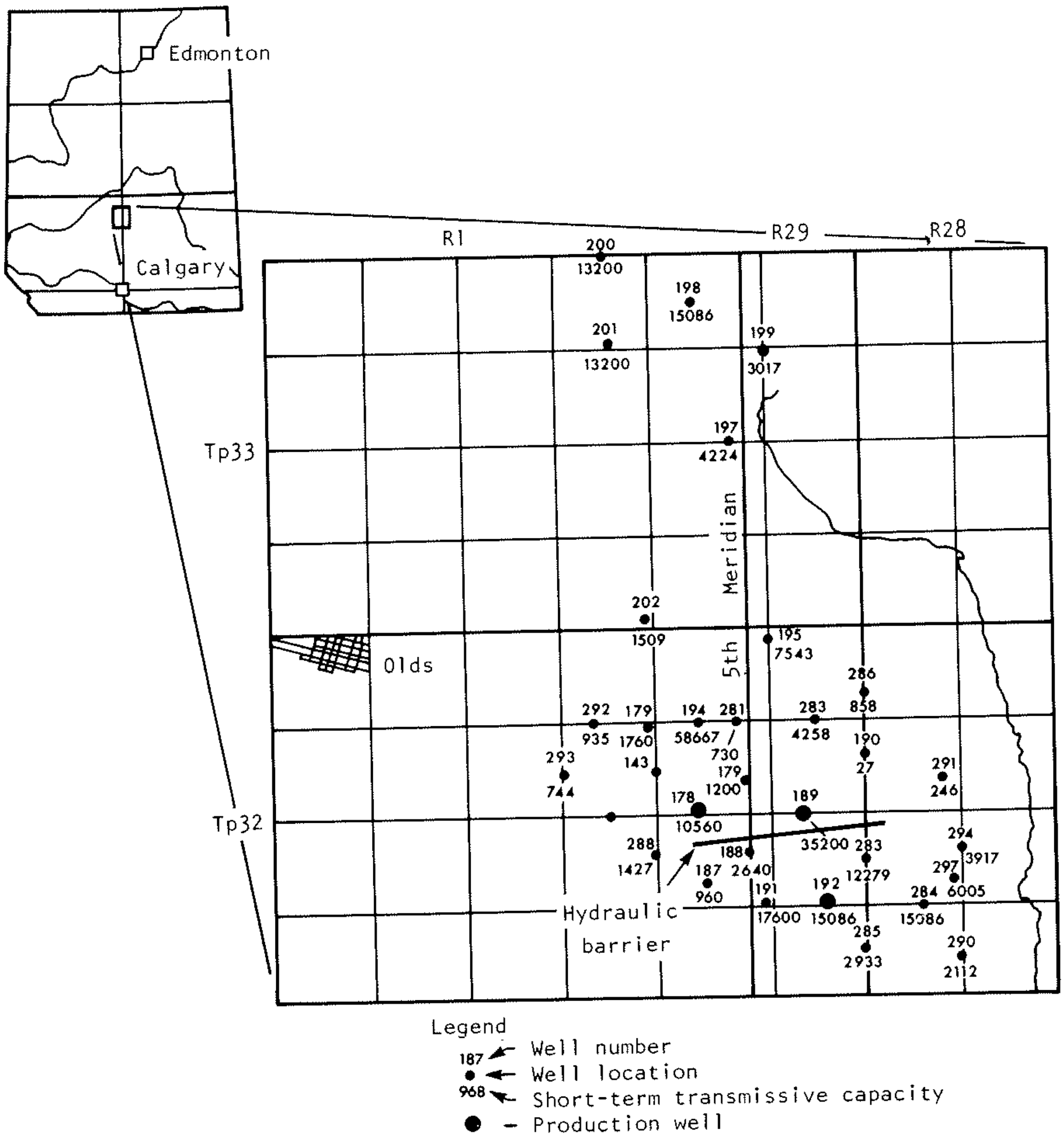


Figure 17. Olds: well locations and short-term transmissive capacities

The results of medium-term pump tests on the three production wells are given in appendix 4.

The water level and production records for the three production wells are plotted on an arithmetic scale in figures 18, 19, and 20. The values plotted are the observed water levels at the end of each month and the monthly volume of pumping. For each well the water level record is complete from the beginning of production in 1965 to the end of 1973, but the production records over the period are incomplete, with relatively large intervals when no records are available and when average values have had to be approximated.

Analysis of the water level and production records to obtain long-term transmissive capacity has been attempted only for well number 192 because this well, being effectively separated from the other pumped wells by the hydraulic barrier, is not influenced by the pumping of the other two wells. Wells 178 and 189 are known to interfere with each other, but the information available is not sufficient to allow assessment of the extent of the interference and compensation for it.

Observed monthly production volumes are available for well 192 for two periods (Fig. 20). During the first period, March 1968 to December 1969, the pumping rate oscillated about an average of 1.69×10^6 igals per month or 38.5 igpm. During the second period, January 1971 to November 1973, the rate fluctuated about an average of 3.64×10^6 igals per month or 82.8 igpm, approximately double that of the first period.

The water level record (Fig. 20) consists of two segments, the first from May 1965 to February 1970 and the second from February 1970 to November 1973. In each of these periods the levels show an exponential decline.

The production and water level records appear therefore to be consistent. The analysis of these records assumes a constant pumping rate of 39 igpm for the period May 1965 to February 1970 followed by a constant rate of 75.4 igpm for the remainder of the period of record. In both cases the pumping rates are the averages of the observed and estimated values during the two periods. With this model for the history of the production of the well, long-term transmissive capacity values can be calculated in the usual way.

The semilog plots of time-drawdown for the two periods are shown in figures 21 and 22. For the first period, the plot is of observed drawdowns. For the second period, the principle of superposition has been applied so that the plot is of differential drawdowns for a pumping rate of (75.4-39.0) or 36.4 igpm, assumed to commence on February 1970. The differential drawdowns are computed by subtracting from the observed drawdowns those drawdowns that would have resulted at a pumping rate of 39.0 igpm.

For the period to February 1970, the drawdown data can be fitted fairly well by a straight line having a slope Δs,

$$\Delta s = 16.94 \text{ ft/log cycle}$$

so that,

$$T = \frac{264Q}{\Delta s} = 607.8 \text{ igpd/ft.}$$

For the period from February 1970 to November 1973 the later differential drawdowns can be fitted by a straight line having a slope

$$\Delta s = 9.6 \text{ ft/log cycle}$$

so that,

$$T = 1001 \text{ igpd/ft.}$$

Averaging these two values gives 800 igpd/ft which can be regarded as the observed long-term transmissive capacity of the well.

Estimates of the parameters of G, A, and H for Olds number 192 calculated from a random sample of 33 short-term transmissive capacities are given in table 7 and figure 23 for various values of m.

The estimates have the same two properties as were observed in the Innisfail case: they decrease monotonically as m increases and except for the first few values of m do not change very much for fairly large changes in m. In fact, the estimates are even less sensitive to changes in m than the ones at Innisfail because there is much less difference between T_1 = 15,086 igpd/ft and $\exp(\hat{\mu}_Y)$ = 3308 igpd/ft. In this respect Olds number 192 is likely to be more typical than Innisfail.

As in the Innisfail case, $\exp(\hat{\mu}_F)$ and $\hat{\alpha}_G$ are similar. For m = 15, $\exp(\hat{\mu}_F)$ estimates the expected value of long-term transmissive capacity to be 5056 igpd/ft with a 95

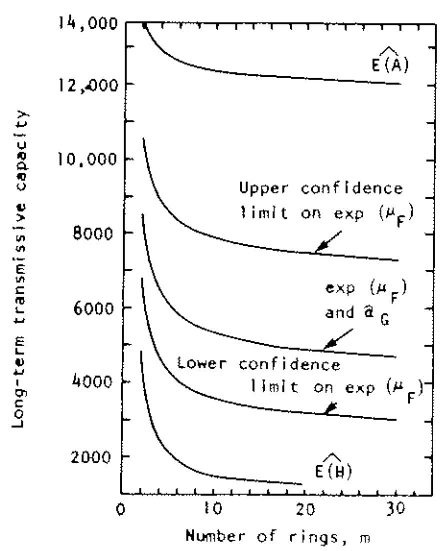


Figure 23. Olds #192: estimates of geometric, arithmetic, and harmonic means

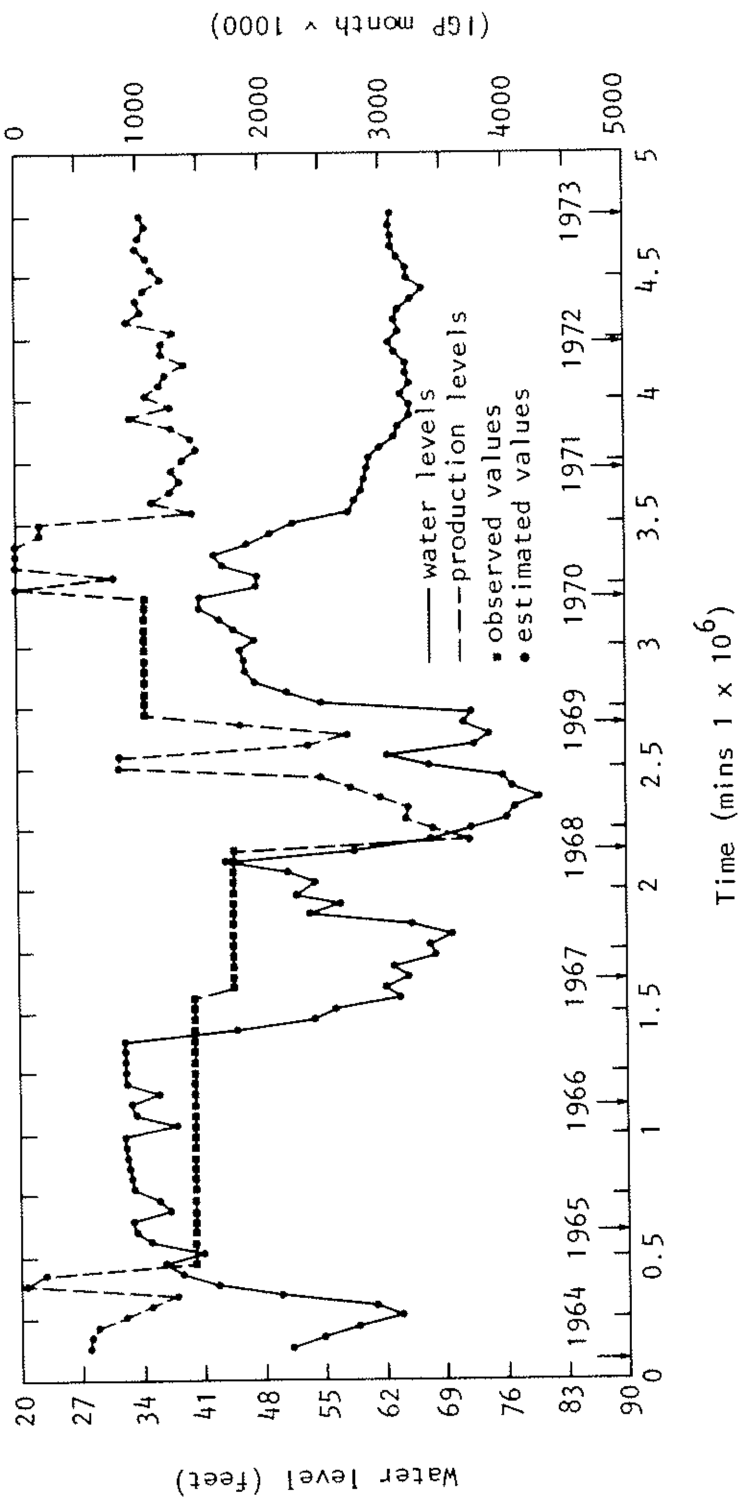


Figure 18. Olds #178: production and water level records

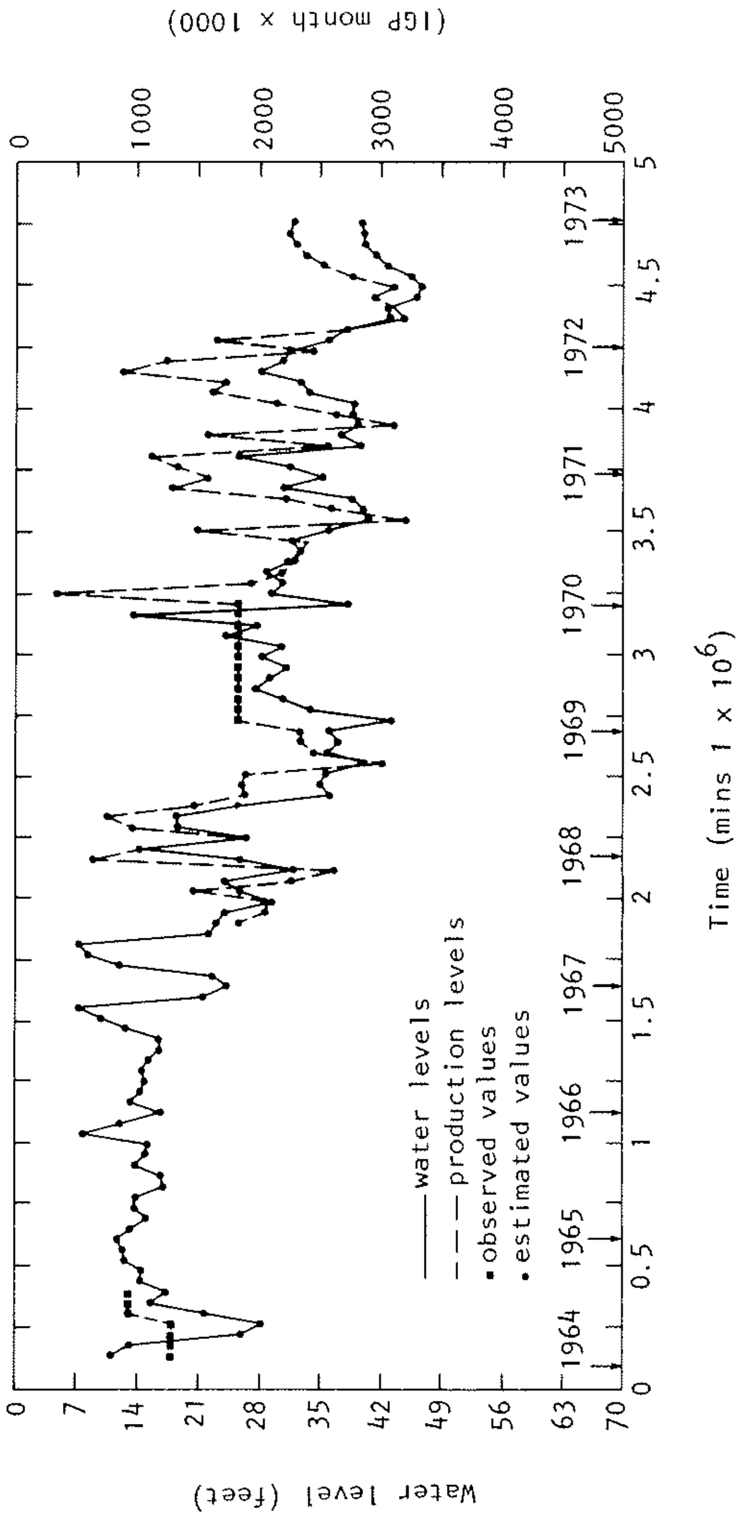


Figure 19. Olds #189: production and water level records

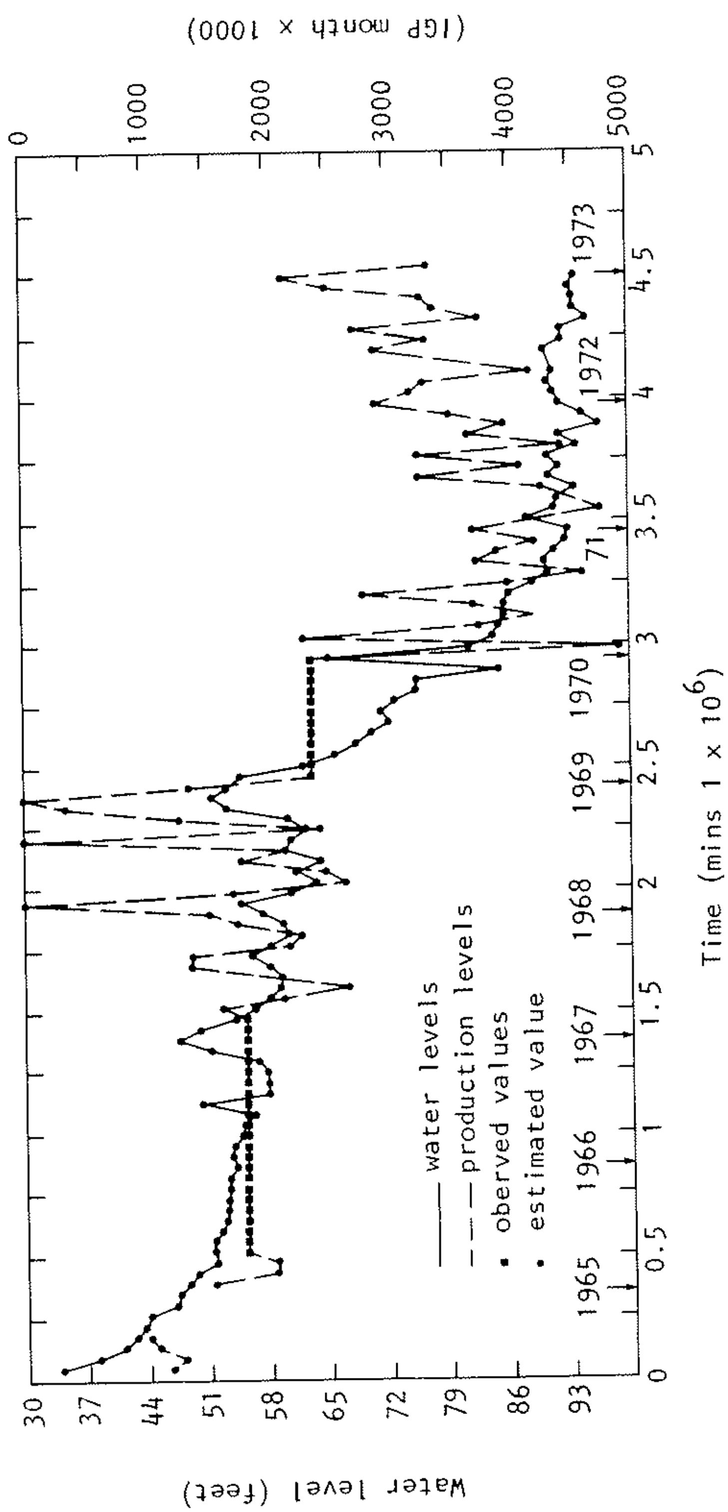


Figure 20. Olds #192: production and water level records

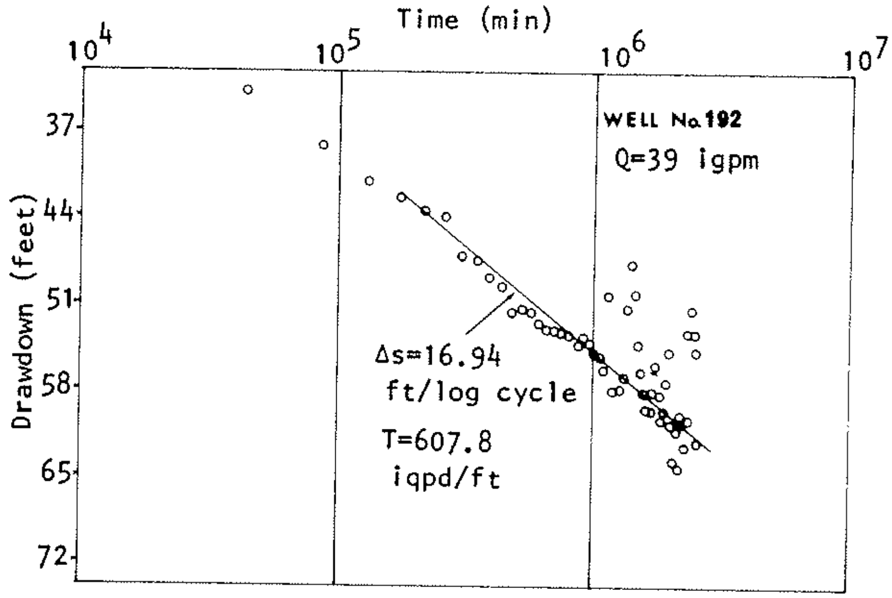


Figure 21. Olds #192: semilog plot of drawdowns to February 1970

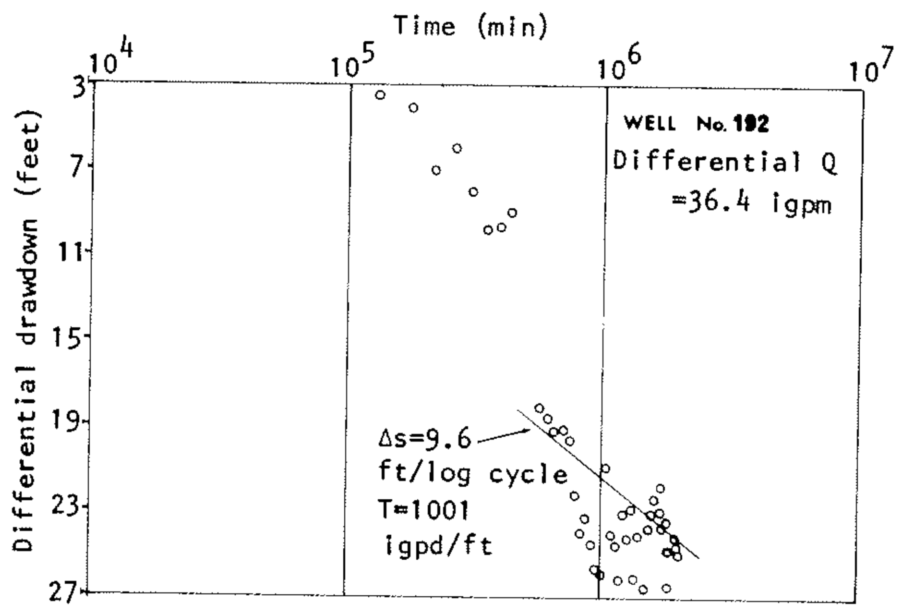


Figure 22. Olds #192: semilog plots of differential drawdowns from February 1970 to November 1973

Table 7. Olds #192: Estimates of Parameters of Geometric, Arithmetic, and Harmonic Means

Number of rings	Number of nodes	$\widehat{E(H)}$[1]	Lower confidence interval on $\exp(\mu_F)$	$\exp(\widehat{\mu_F})$[2]	Upper confidence interval on $\exp(\mu_F)$	$\widehat{\alpha}_G$[3]	$\widehat{E(A)}$[4]	$\sqrt{\widehat{Var(H)}}$[5]	$\widehat{\sigma}_F$[6]	$\widehat{\beta}_G$[7]	$\sqrt{\widehat{Var(A)}}$[8]
1	1	-	-	15086	-	15086	-	-	-	-	-
2	4	4881	6892	8539	10581	8974	13605	3883	.35	3180	6309
3	9	3127	5351	7106	9436	7321	13127	1846	.31	2300	5683
4	16	2455	4698	6465	8896	6510	12881	1335	.29	1860	5158
5	25	1913	4330	6093	8574	6135	12727	744	.26	1600	4785
6	36	1700	4090	5846	8356	5847	12619	727	.25	1430	4494
7	49	1786	3920	5668	8196	5641	12539	640	.24	1310	4278
8	64	1638	3791	5533	8073	5485	12476	699	.23	1220	4099
9	81	1589	3690	5425	7975	5361	12425	600	.22	1140	3950
10	100	1543	3608	5337	7894	5260	12382	634	.21	1090	3821
11	121	1581	3539	5263	7826	5176	12346	403	.21	1040	3715
12	144	1439	3481	5200	7768	5104	12315	517	.20	996	3633
13	169	1443	3431	5146	7717	5042	12287	464	.20	961	3550
14	196	1565	3387	5098	7672	4988	12263	352	.19	930	3479
15	225	1433	3349	5056	7632	4940	12241	349	.19	903	3406
16	256	1345	3314	5018	7597	4898	12222	457	.19	880	3347
17	289	1342	3283	4984	7565	4859	12204	386	.18	858	3302
18	324	1281	3255	4953	7535	4824	12188	387	.18	839	3241
19	361	1331	3230	4925	7508	4793	12173	481	.18	821	3194
20	400	1323	3207	4899	7484	4764	12159	393	.17	805	3158
30	900	-	3046	4719	7312	4563	12062	-	.16	696	2862
40	1600	-	2953	4615	7210	4447	12004	-	.15	634	2681
50	2500	-	2891	4544	7141	4369	11963	-	.14	593	2557
60	3600	-	2845	4491	7089	4311	11933	-	.14	563	2464
70	4900	-	2809	4450	7049	4266	11909	-	.13	540	2390
80	6400	-	2780	4416	7016	4229	11890	-	.13	521	2328
90	8100	-	2756	4388	6988	4199	11873	-	.13	505	2278
100	10000	-	2735	4365	6964	4173	11859	-	.12	492	2234
200	40000	-	2620	4231	6830	4027	11778	-	.11	421	1982
300	90000	-	2566	4167	6766	3959	11738	-	.10	388	1860
400	160000	-	2532	4127	6725	3915	11713	-	.099	367	1783
500	250000	-	2508	4098	6696	3885	11695	-	.095	352	1726
∞	-	-	-	3308	-	3308	-	-	-	-	-

All values of estimators are in igpd/ft

T_I = 15086 igpd/ft

$\widehat{\mu}_Y$ = 8.10

$\widehat{\sigma}_Y$ = 1.61

Sample size = 33

$t_{32,.95}$ = 2.04

[1] Estimate of lower bound on possible values of long-term transmissive capacity

[2] Estimate of expected value of long-term transmissive capacity

[3] Estimate of mean of G

[4] Estimate of upper bound on possible values of long-term transmissive capacity

[5] Estimate of standard deviation of H

[6] Estimate of standard deviation of F

[7] Estimate of standard deviation of G

[8] Estimate of standard deviation of A

Table 8. Olds #192: Values of 20-year Safe Yields Based on Estimates of Long-term Transmissive Capacity

Number of rings	$\widehat{E(H)}$ [1]	Lower confidence interval on $\exp(\mu_F)$	$\exp(\widehat{\mu_F})$ [2]	Upper confidence interval on $\exp(\mu_F)$	$\widehat{\alpha_G}$ [3]	$\widehat{E(A)}$ [4]
1	-	-	-	-	-	-
2	337	399	437	474	446	514
3	259	354	405	455	410	508
4	220	330	388	445	391	505
5	184	315	377	438	378	503
6	168	305	370	434	370	502
7	175	298	364	430	363	501
8	163	292	360	428	358	500
9	160	287	356	425	354	500
10	156	283	353	424	350	499
11	159	280	350	422	347	499
12	148	277	348	421	345	498
13	148	275	346	420	343	498
14	158	272	345	419	341	498
15	147	270	343	418	339	497
16	140	269	342	417	337	497
17	140	267	341	416	336	497
18	134	266	340	415	335	497
19	139	264	338	415	334	496
20	138	263	337	414	332	496
30	-	255	331	410	325	495
40	-	249	327	408	320	494
50	-	246	324	406	317	494
60	-	243	322	404	315	493
70	-	241	320	403	313	493
80	-	240	319	403	311	493
90	-	238	318	402	310	493
100	-	237	317	401	309	492
200	-	230	311	398	302	491
300	-	227	309	396	299	491
400	-	225	307	395	298	490
500	-	223	306	394	296	490

Available Drawdown = 110 ft
t = 5760 mins
S_t = 23 ft
Q = 152 igpm

$$Q_{20} = \frac{Ad}{\frac{S_t}{Q} + \frac{264}{T_L}(7 - \log t)} \text{ igpm}$$

[1] Estimate of lower bound on possible values of long-term transmissive capacity
[2] Estimate of expected value of long-term transmissive capacity
[3] Estimate of mean of G
[4] Estimate of upper bound on possible values of long-term transmissive capacity

percent confidence interval of 3349 to 7632 igpd/ft. The estimate on the lower bound on possible values of long-term transmissive capacity is $\hat{E}(H)$ = 1433 igpd/ft and the upper bound is $\hat{E}(A)$ = 12,241 igpd/ft. Choosing estimates of the expected value of long-term transmissive capacity and upper and lower bounds on possible values for any value of m in the interval m = 8 to m = 200 does not change these estimates by more than ±15 percent. The smaller difference between T_1 and exp(μ) than at Innisfail makes $\hat{E}(H)$ and $\hat{E}(A)$ practically useful bounds on possible values of long-term transmissive capacity.

For calculating 20-year safe yields corresponding to the estimates of long-term transmissive capacity the medium-term pump test on well number 192 (Fig. 20) was used to give s_t = 23 ft at t = 5760 min for Q = 152 igpm. The available drawdown for the well, calculated as the difference between the static water level prior to production and the top of the first producing zone, was taken as 110 ft. These data as well as the values of 20-year safe yield are given in table 8. For m = 15, the expected 20-year safe yield is 343 igpm, with a 95 percent confidence interval of 270 to 418 igpm. Lower and upper bounds on possible values are 147 to 497 igpm.

Thus, it is seen at Olds number 192 that the short-term transmissive capacity of 15,086 igpd/ft decreases to a medium-term value of 5900 igpd/ft and, in the absence of a barrier, to an expected long-term value of 5056 igpd/ft. The barrier, however, has the effect of decreasing the observed long-term value of 800 igpd/ft, which is even less than the estimate of lower bound on possible values. The effect of contravening an implicit premise of the method of estimation — that the region should be completely hydraulically connected — is clearly demonstrated.

The difference between a barrier, to which the method is not appropriate, and an area of low transmissive capacity, to which it is, should be understood. A barrier is a plane of zero permeability and essentially zero width. It causes hydraulic discontinuity and is really a boundary and not part of the flow regime. An area of low transmissive capacity is an integral part of the flow regime and can be sampled by a pump test. As such it can be taken into account by the estimator.

PREDICTING DRAWDOWN, CAUSED SOLELY BY PUMPING, AT A WELL IN A HETEROGENEOUS AQUIFER

The method of estimating 20-year safe yields gives a single value of sustainable yield which is valid if the well is pumped continuously at this constant rate throughout the 20-year period. Such a single value is useful, for example, for assessing the resources of an area or as a basis for comparing different wells and areas. In practice, specific wells are produced at variable rates, as at Innisfail (Fig. 14) and Olds (Figs. 18, 19, and 20). This type of situation requires that the method of estimation be modified to take the variability into account. This is done by assuming that linearity exists between the drawdown and pumping rate and that the principle of superposition can be applied to drawdowns.

The principle of linearity between drawdown and pumping rate has already been tacitly assumed both in present methods of analyzing pumping tests in heterogeneous strata in Alberta (Fig. 8) and in the method for estimating 20-year safe yields which has been described (Fig. 12). The principle can be explained as follows:

If a well is pump-tested at a constant pumping rate, Q, for a time period, D, the drawdown curve is as shown in figure 24. Now if $S_{d,Q}$ is the drawdown at time d for $0 \leq d \leq D$, then at any other constant pumping rate, Q_1, the principle of linearity allows the drawdown at time d to be calculated as,

$$S_{d,Q_1} = \left(\frac{Q_1}{Q}\right) S_{d,Q} \quad 0 \leq d \leq D.$$

This principle is observed to be valid in practice in Alberta when a well in heterogeneous media is pumped at different rates.

The principle of superposition of drawdowns can be explained as follows:

If the drawdown at time d, $0 \leq d \leq D$, resulting from pumping at rate Q is $S_{d,Q}$, then, for a production program

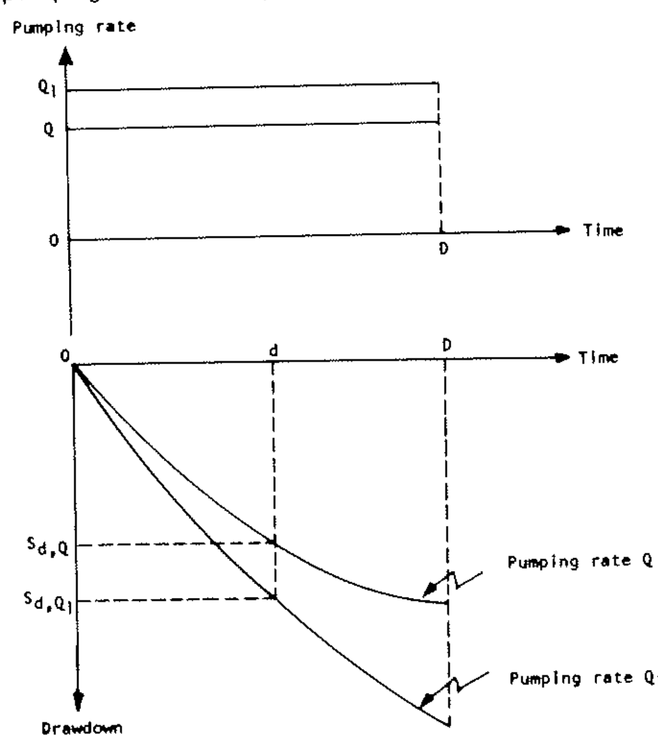


Figure 24. Illustration of the principle of linearity between drawdown and pumping rates

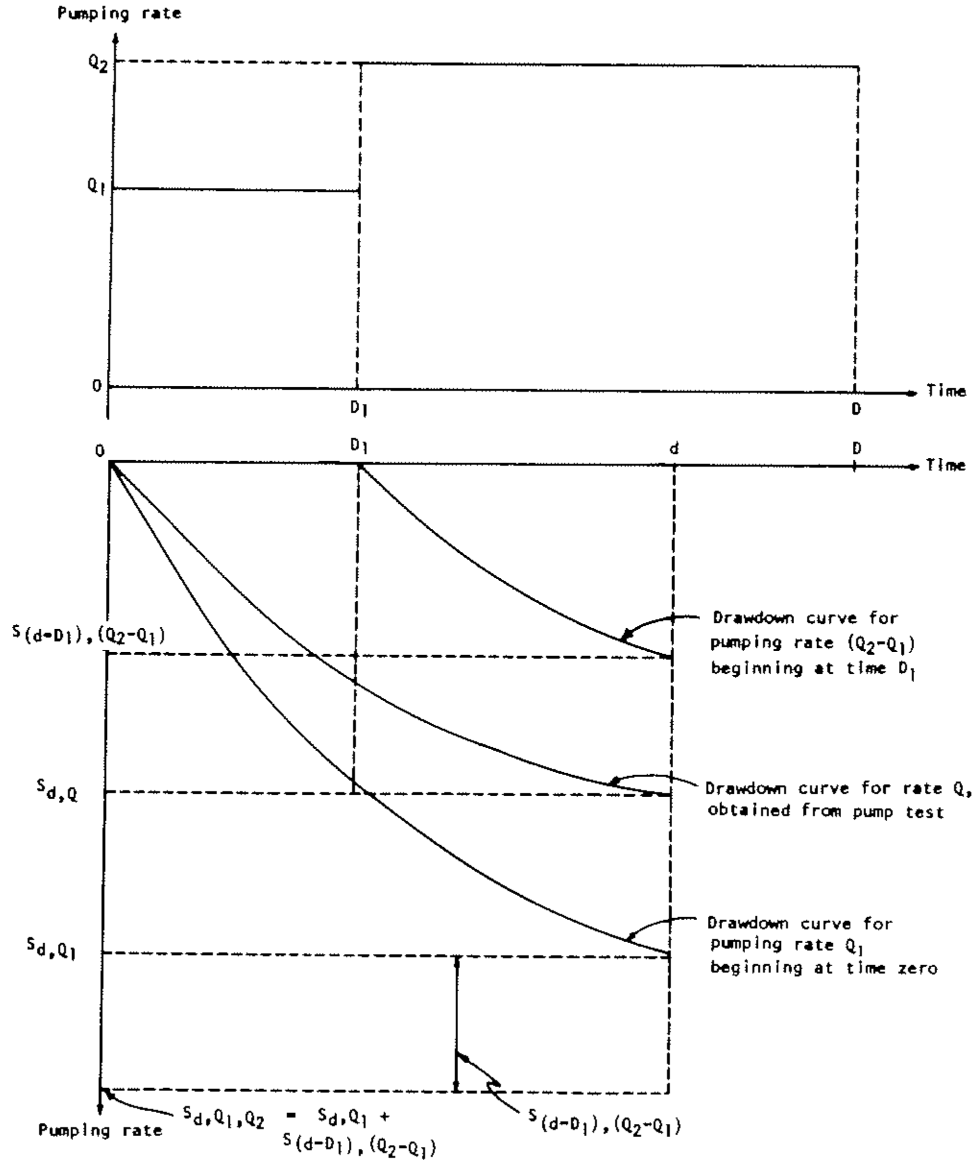


Figure 25. Illustration of the principle of superposition

consisting of pumping at rate Q_1 for time 0 to D_1 and Q_2 for D_1 to D (Fig. 25), the drawdown at time d, $D_1 \leq d \leq D$, is, by the principle of superposition:

$$S_{d,Q_1,Q_2} = S_{d,Q_1} + S_{(d-D_1),(Q_2-Q_1)}$$

which, by the principles of linearity, becomes,

$$S_{d,Q_1,Q_2} = \left(\frac{Q_1}{Q}\right) S_{d,Q} + \left(\frac{Q_2-Q_1}{Q}\right) S_{(d-D_1),Q} .$$

This is shown graphically in figure 25. In words, the initial pumping rate, Q_1, is imagined to continue unchanged beyond time D_1 and a pumping rate (Q_2-Q_1) is imagined to commence at time D_1. To obtain the drawdown at time d caused by the real production program, the drawdowns caused independently by these two imaginary pumping rates are simply added.

In the general case, given a time-drawdown curve for any constant pumping rate Q and duration D, the drawdown resulting from pumping at an arbitrary number of rates, (Q_i, i = 1 . . . n), for periods of time (D_i-D_{i-1}) is (Fig. 26),

$$S_d = \frac{1}{Q} \sum_{i=1}^{n} (Q_i-Q_{i-1}) \left[S_{(d-D_{i-1}),Q} \right]$$

for

$Q_o = 0$

$D_o = 0$

$D_{n-1} \leq d \leq D$.

It will be noted that it is not possible to calculate drawdowns resulting from arbitrary production programs for times greater in duration than the pump test, D, unless one has knowledge of the drawdown curve at these later times.

The principle of superposition is valid where the partial differential equation describing flow in the system is linear, which is true if transmissivity is not a function of head. This can be considered the case.

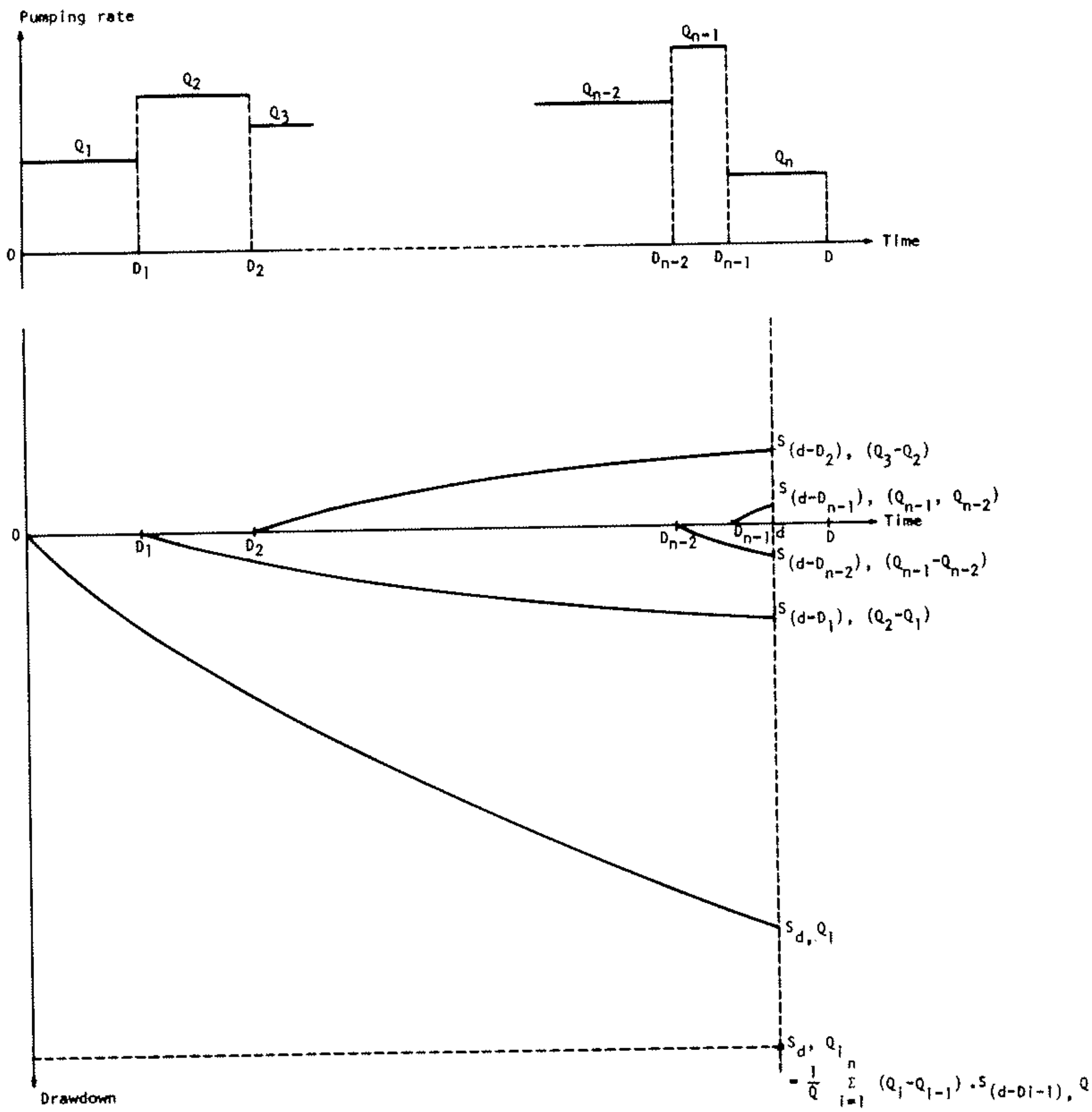


Figure 26. Principle of superposition used to compute drawdown resulting from an n-period production program

ADAPTATION OF THE METHOD TO THE SITUATION IN WHICH THE SEMILOG PLOT OF THE TIME-DRAWDOWN CURVE CONSISTS OF A SERIES OF STRAIGHT-LINE SEGMENTS

Commonly, when plotted on semilog paper, pumping tests conducted in heterogeneous media consist of a series of straight-line segments. If the drawdown per log cycle of each of the segments is Δs_i, $i = 1 \ldots N$ for pumping at a constant rate Q (Fig. 27) and the times at which the changes in slope occur are δ_i, $i = 1 \ldots N$, then the drawdown at time d resulting from production at successive constant rates Q_i, $i = 1 \ldots r$ for periods of time $(D_{i-1}, D_i, i = 1 \ldots r)$, where $D_{j-1} \leq d \leq D_j$, $j = 1 \ldots r$, is

$$s_d = \sum_{p=1}^{n} \sum_{i=q(p)+1}^{q(p+1)} \left(\frac{Q_i - Q_{i-1}}{Q}\right) \left[\sum_{m=1}^{n-p} \Delta s_m \log\left(\frac{\delta_m}{\delta_{m-1}}\right) + \Delta s_{n+1-p} \log\left(\frac{d - D_{i-1}}{\delta_{n-p}}\right) \right]$$

where,

n = number of straight-line segments up to time d; $1 \leq n \leq N$

$q(1) = 0$

$q(p)$ = last pumping interval to generate a curve with $(n+2-p)$ slopes before time d

$q(n+1) = j$

$Q_o = 0$

$\delta_o = 1$

$$\sum_{m=1}^{0} = 0.$$

A computer program to evaluate this equation is given in appendix 5.

For the instance of there being just two slopes (N = 2) the equation becomes,

$$s_d = \sum_{i=1}^{q(2)} \left(\frac{Q_i - Q_{i-1}}{Q}\right) \left[\Delta s_1 \log(\delta_1) + \Delta s_2 \log\left(\frac{d - D_{i-1}}{\delta_1}\right) \right]$$

$$+ \sum_{i=q(2)+1}^{j} \left(\frac{Q_i - Q_{i-1}}{Q}\right) \left[\Delta s_1 \log(d - D_{i-1}) \right] \quad \text{for } n = 2$$

and

$$s_d = \sum_{i=1}^{j} \left(\frac{Q_i - Q_{i-1}}{Q}\right) \left[\Delta s_1 \log(d - D_{i-1}) \right] \quad \text{for } n = 1.$$

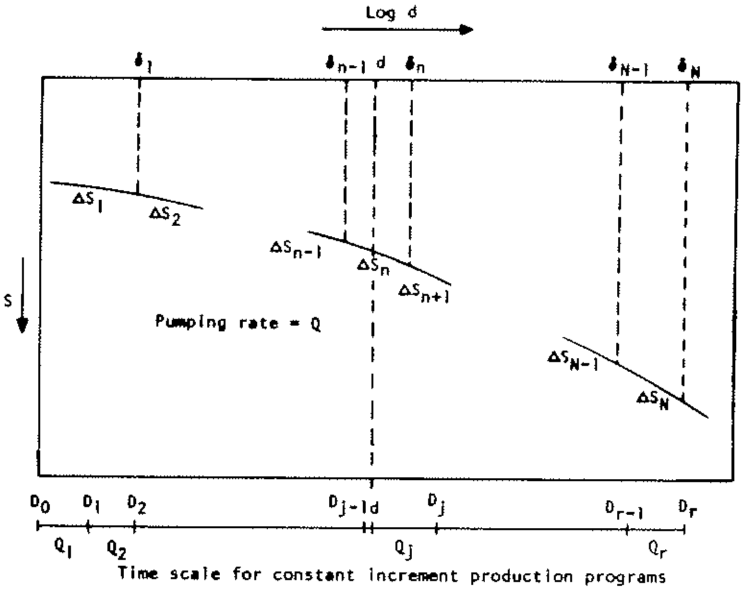


Figure 27. Diagrammatic semilog plot of the time-drawdown curve

FIELD APPLICATION OF THE METHOD: INNISFAIL

The location and hydrogeological setting of the water-supply well at Innisfail have been previously described and graphs of water level records, production records, and short-term pump tests are given in appendix 3 and figures 14 and 15. The availability of long-term water level and production records, and the fact that it is not subject to any interference, makes this well the best available case on which to do a history-match to demonstrate the method, and to make predictions of future drawdowns by using the estimate of the expected long-term transmissive capacity.

From the short-term pump test (appendix 3) the slope of the drawdown curve up to 1000 min for Q = 99 igpm is 0.5 ft per log cycle. From the long-term water level record (Fig. 15) the slope of the drawdown curve from 1×10^5 min to 4.56×10^6 min for Q = 99 igpm is 16.25 ft per log cycle. No direct information is available on the drawdown curve for the period from 1×10^3 min to 1×10^5 min. During this period the water level drops from 1.5 ft to 8 ft for Q = 99 igpm. Since this is quite small it will be assumed that the slope of the curve over these two log cycles is a straight line with slope 3.25 ft per log cycle for Q = 99 igpm.

Therefore, at Innisfail, the observed drawdown curve at a constant pumping rate is presumed to consist of three straight-line segments on a semilog scale. The computer program in appendix 5 is then used to generate a water level history at the well for comparison with the observed record. In order to do this, the production record was assumed to consist of a sequence of constant monthly pumping rates, except for periods of non-pumping, which were incorporated exactly (Fig. 14).

The match between the synthesized and observed water level records is shown in figures 28 and 29 on arithmetic and semilog scales and is essentially exact.

To make predictions of drawdowns at the well for production programs that might be followed, the slope of the drawdown curve for Q = 99 igpm is needed for times greater than 4.56×10^6 min. This of course is obtained from the estimate of the expected value of long-term transmissive capacity. As already described this estimate is essentially the same as the observed value of long-term transmissive capacity so that the slope beyond 4.56×10^6 min will continue to be 16.25 ft per log cycle.

The drawdowns resulting from the following production programs were generated for a period of 10 years beyond the existing record:

(1) $Q = 19{,}021 + 3.3031\,t$ ft^3/day for t in days
(2) Q = 200 igpm
(3) as (1) + 10,000 igpd
(4) as (1) + 500,000 igpd
(5) Q = 0.

The results of these production programs are plotted on figures 30 and 31.

Production program (1) is that obtained by extrapolating the equation defining the increase in production over the past eight years (Fig. 14). It is therefore a simple continuation of the historic trend in production. Program (2) is that resulting from pumping continuously at the rate which is known to be safe for the well. (It is not known if the well, completed open hole, will cave in if pumped at rates much greater than 200 igpm). Program (3) is the same as (1) with an imposed industrial demand of 10,000 igpd. Program (4) is the same as (1) with an imposed industrial demand of 500,000 igpd. Program (5) is simply the recovery of water levels if production ceases. Programs (1), (3), and (4) are realistic alternatives based on situations presently being considered by the town. The other two programs are included for academic interest.

Programs (1), (2), and (3) are clearly easily within the capacity of the well for the next 10 years, bearing in mind that the available drawdown is to 80 ft and that the maximum pumping rate for any of the programs is 210 igpm. Program (4) is clearly beyond the capacity of the well.

In the more typical case when predictions of future drawdowns have to be made prior to any production, the same approach would be taken. It may be usual that only a medium-term test is available for determining the slopes of the drawdown curve at a constant pumping rate and that for time periods greater than the duration of the test the slope has to be obtained from the estimate of the expected value of long-term transmissive capacity.

The method therefore provides a simple and rapid means for assessing various production program options and assigning bounds to estimates of drawdowns resulting from these programs.

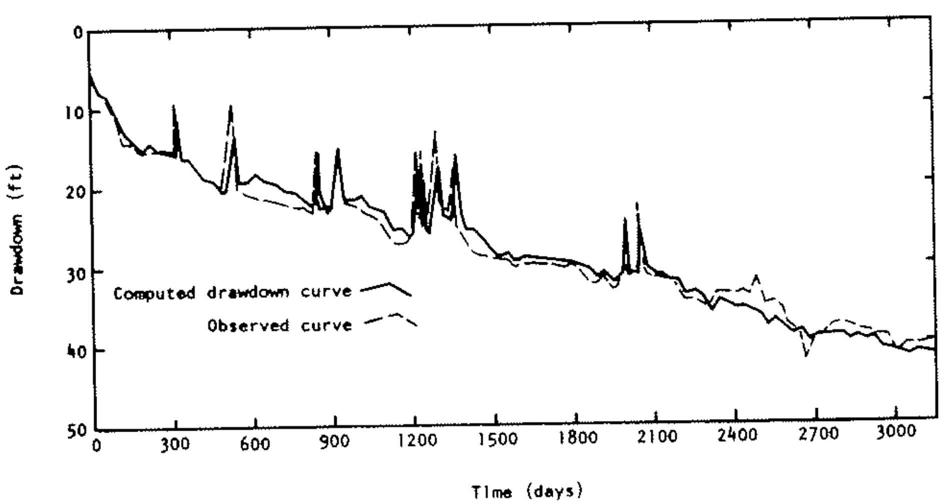


Figure 28. Innisfail: comparison of observed and computed drawdown curves for observation well (arithmetic scale)

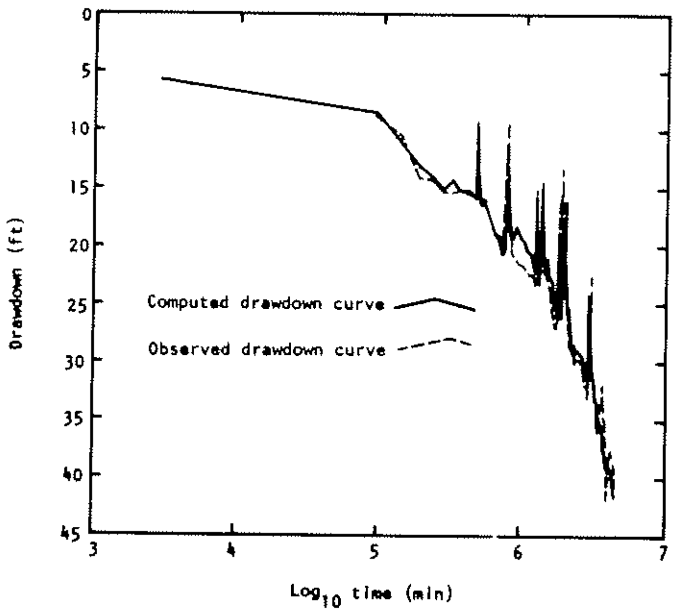


Figure 29. Innisfail: comparison of observed and computed drawdown curves for observation well (semilog scale)

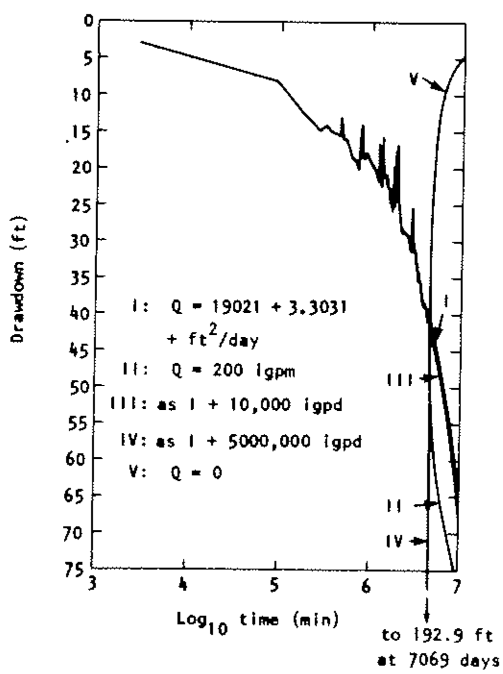


Figure 30. Innisfail: predicted drawdowns at well for different production programs over next 10 years (semilog scale)

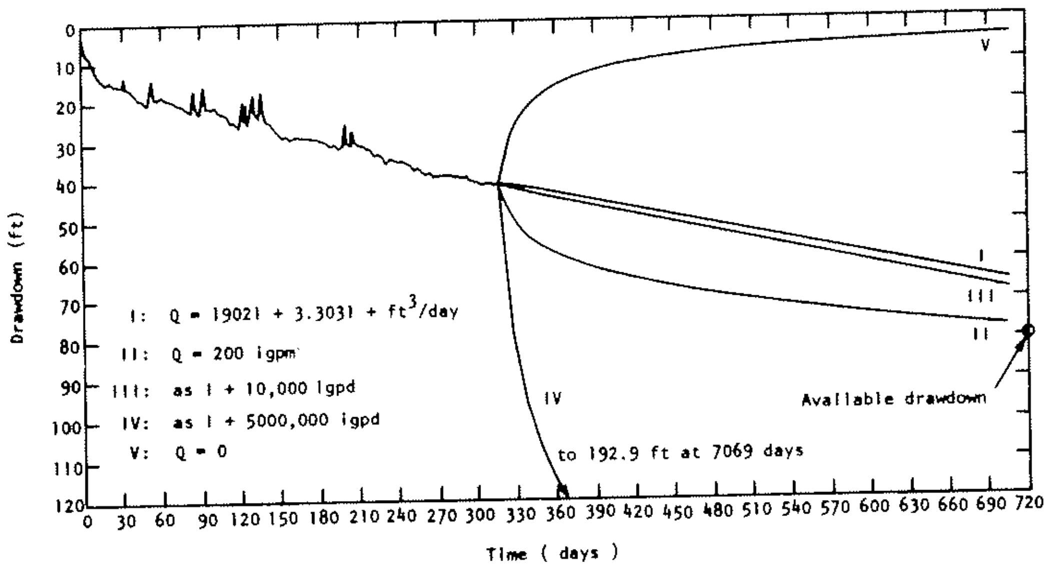


Figure 31. Innisfail: predicted drawdowns as well for different production programs over next 10 years (arithmetic scale)

SUMMARY

The prediction of sustainable yields to wells completed in heterogeneous aquifers in Alberta cannot be made by any existing method of analysis. Such a high degree of heterogeneity exists that it is not possible to accommodate the aquifers within the very simple models that are of necessity adopted by such methods.

In devising a means of predicting sustainable yields to individual wells, it has been recognized that the only relevant information on the hydraulic nature of the aquifer is the drawdown curves obtained from pump tests. If the response of a well to pumping at a constant rate is known, it is possible, using the principle of linearity, to predict the well's response at other rates. However, because of the heterogeneous nature of the aquifer, such predictions cannot be made for times greater than the duration of the pump test. Since it is only practical to conduct pump tests for periods of several days, the drawdown curve at greater times must be estimated.

A method has been described for making such an estimate of long-term transmissive capacity and using it to predict 20-year safe yields and the drawdown at the well resulting from arbitrary long-term production programs.

The estimates of long-term transmissive capacity are made using a random sample of short-term transmissive capacity values. There are several good reasons for the use of such values as the basis of the estimate. Both short-term and long-term transmissive capacities are determined by the same physical process and differ only in scale. The short-term values measure portions of the aquifer which are included in the determination of the long-term values. It is to be expected, therefore, that the long-term value is some average of the sample of short-term values: the issue is whether or not the sample of short-term values is representative and the proposed method of averaging is the correct one. The short-term transmissive capacities are relatively easily obtained, which is a necessary attribute if a random sample of them is required. In fact, they are usually obtained in an hydrogeologic investigation.

The estimators of long-term transmissive capacity are empirical and therefore their general validity has to be demonstrated by application to field cases. The extent to which this has been possible with cases in Alberta is insufficient to constitute thorough verification of the method. The only field case which is adequately documented and meets the assumptions of the method is at Innisfail. It is an extreme test of the method. The very good correlation that is obtained between the observed and estimated values of long-term transmissive capacity is strong evidence of the validity of the method, but further tests are needed.

The method that has been described for estimating drawdown resulting from an arbitrary production program should prove a very useful tool in evaluating any

number of options for the management of the well. This method is basically independent of the method of estimating long-term transmissive capacity. It only requires knowledge of the drawdown curve for 20 years at some constant rate, no matter how the knowledge is obtained. The method of estimating long-term transmissive capacity is just one way, but hopefully the best, of obtaining this knowledge.

It is true that this method of estimating long-term transmissive capacity is a lumped-parameter approach; the detailed mechanics of groundwater flow in the aquifer are ignored. Everything is lumped at a point, the well, where excitation and response of the system are measured. In fact, it is because the method is lumped-parameter that situations such as Olds, where a distinct boundary exists, cannot be evaluated by this method. However, the conventional distributed-parameter approach in which the details of flow are modeled in space and time is not viable in complex heterogeneous systems except perhaps on a gross scale. Having to adopt a lumped-parameter approach and forego detailed knowledge of system behavior can simply be regarded as the price that has to be paid in heterogeneous systems in order to have tools that are useful in practice.

SUGGESTIONS FOR FURTHER RESEARCH

The methods described in this report allow the drawdown resulting from arbitrary production programs to be estimated, but only at the production well. An extension of this research would be to devise means of estimating drawdowns at points in the aquifer away from the production well. In this way, the effects of interference in well fields could be taken into account. The problem is far from trivial, as is shown by figure 6.

In this report heterogeneity has been described in qualitative terms. This is obviously not completely satisfactory and means of quantifying heterogeneity would be very useful. The concept of entropy as used in Information Theory (Domenico, 1972) is a possible means of doing this. This theory also has the potential for use in determining how much information to collect and in assessing the worth of the next observation in sampling.

The method of estimating long-term transmissive capacity was devised primarily as an intermediate but essential step in predicting sustainable yields. However, it is not restricted to this use. For example, it would constitute a very important parameter in mapping groundwater resources of heterogeneous sediments.

As shown in figure 4, drawdown curves for short-term pump tests in heterogeneous strata have a variety of shapes, reflecting the very complex hydraulic response of the aquifer to pumping. A closer investigation of these pump tests would help to explain the detailed nature of this response.

RECOMMENDATION

The estimators that have been derived for estimating long-term transmissive capacity are empirical and therefore have to be validated by field examples. There is in Alberta a dearth of the kind of data needed for this purpose, especially records of long-term water levels and production. Such data need to be collected, especially since for any supply well this type of information is necessary if the well is to be properly managed.

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APPENDIX 1
DERIVATION OF MINIMUM VARIANCE UNBIASED ESTIMATORS OF α_G, β^2_G

Consider the random variable $Y \sim N(\mu_y, \sigma_y^2)$ and maximum likelihood estimators,

$$\hat{\mu}_y = \frac{1}{t} \sum_{i=1}^{t} y_i$$

$$\hat{\sigma}_y^2 = \frac{1}{t-1} \sum_{i=1}^{t} (y_i - \hat{\mu}_y)^2$$

where $\hat{\mu}_y$ and $\hat{\sigma}_y^2$ are independent and jointly sufficient.

It is desired to find the minimum variance unbiased estimator of

$$\Theta = \exp\{C_1 + C_2 \mu_y + C_3 \sigma_y^2\}.$$

Now,

$$\hat{\mu}_y \sim N(\mu, \frac{\sigma_y^2}{t})$$

therefore,

$$E(\exp\{C_1 + C_2 \hat{\mu}_y\}) =$$

$$\int_{-\infty}^{\infty} \left(e^{C_1 + C_2 \hat{\mu}_y} \right) \left(\frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_y / \sqrt{t}} \right) \left(\exp\{-\frac{1}{2\sigma^2/t} (\hat{\mu}_y - \mu_y)^2\} \, d\hat{\mu}_y \right)$$

$$= e^{C_1} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\, \sigma_y / \sqrt{t}}$$

$$\left(\exp\{-\frac{1}{2\sigma_y^2/t} (\hat{\mu}_y^2 - 2\hat{\mu}_y (\mu_y + \frac{C_2 \sigma_y^2}{t}) + \mu_y^2)\} \, d\hat{\mu}_y \right)$$

$$= e^{C_1} \left(\exp\{\mu_y C_2 + \frac{C_2^2 \sigma_y^2}{t}\} \right)$$

$$\left(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\, \sigma_y / \sqrt{t}} \exp\{-\frac{1}{2\sigma_y^2/t} (\hat{\mu}_y - \mu_y + C_2 \frac{\sigma_y^2}{t})^2\} \, d\hat{\mu}_y \right)$$

$$= \exp\{C_1 + \mu_y C_2 + C_2^2 \frac{\sigma_y^2}{t}\}$$

$$= \Theta \left(\exp\{-\sigma_y^2 \cdot \frac{(tC_3 - C_2^2)}{t}\} \right).$$

Now, (Aitchison and Brown, 1957):

$$E(\hat{\sigma}_y^{2j}) = \frac{(t-1)(t+1) \ldots\ldots (t-3+2j)}{(t-1)^j} \left(\sigma^{2j} \right), \; j = 1, 2 \ldots\ldots$$

let,

$$\Psi_t(S) = 1 + S + \frac{1}{2!} \frac{(t-1)}{(t+1)} S^2 + \frac{1}{3!} \frac{(t-1)^2}{(t+1)(t+3)} S^3 + \ldots.$$

and since,

$$e^S = 1 + S + \frac{S^2}{2!} + \frac{S^3}{3!} + \ldots..$$

then,

$$E\left(\Psi_t\left[\frac{tC_3 - C_2^2}{t} \hat{\sigma}_y^2\right]\right) = \exp\left\{\frac{tC_3 - C_2^2}{t} \left(\sigma_y^2\right)\right\}$$

therefore,

$$E\left[\exp\{C_1 + C_2 \hat{\mu}_y\} \cdot \Psi_t\left[\frac{(tC_3 - C_2^2)}{t} \left(\hat{\sigma}_y^2\right)\right]\right] = \Theta$$

so that,

$$\hat{\Theta} = \exp\{C_1 + C_2 \hat{\mu}_y\} \left\{\Psi_t\left[\frac{(tC_3 - C_2^2)}{t} \hat{\sigma}_y^2\right]\right\}$$

where, $\hat{\Theta}$ is minimum variance unbiased estimate of Θ.

Specifically,

$$\hat{\alpha}_G = \exp\{C_1 + C_2 \mu_y + C_3 \sigma_y^2\}$$

where,

$$C_1 = \frac{w_1 \log X_1}{\sum_{i=1}^{n} w_i}$$

$$C_2 = \frac{\sum_{j=2}^{n} w_j}{\sum_{i=1}^{n} w_i} \qquad C_3 = \frac{\sum_{j=2}^{n} w_j^2}{2\left[\sum_{i=1}^{n} w_i\right]^2}$$

so that,

$$\hat{\alpha}_G = \hat{\Theta}(C_1, C_2, C_3)$$

and

$$\beta_G^2 = \exp\{2C_1 + 2C_2 \mu_y + 4C_3 \sigma_y^2\} - \exp\{2C_1 + 2C_2 \mu_y + 2C_3 \sigma_y^2\}$$

so that,

$$\hat{\beta}_G^2 = \hat{\Theta}(2C_1, 2C_2, 4C_3) - \hat{\Theta}(2C_1, 2C_2, 2C_3).$$

APPENDIX 2

SUMMARY OF THE METHOD OF ESTIMATING LONG-TERM TRANSMISSIVE CAPACITY

The method of estimating the expected value of long-term transmissive capacity and upper and lower bounds on possible values is summarized below.

STEP 1

Obtain a random sample of short-term transmissive capacity values, τ_i, $i = 1 \ldots t$ and set $T_1 = \tau_1$.

STEP 2

Calculate,

$$\hat{\mu}_Y = \frac{1}{t} \sum_{i=1}^{t} \log \tau_i$$

$$\hat{\sigma}_Y^2 = \frac{1}{t-1} \sum_{i=1}^{t} (\log \tau_i - \hat{\mu}_Y)^2$$

STEP 3

Using table 1 to get values of C_1, C_2, and C_3, calculate,

(1) $\exp(\hat{\mu}_F)$ where,

$$\hat{\mu}_F = C_1 \log T_1 + C_2 \hat{\mu}_Y$$

This is the estimate of the expected value of long-term transmissive capacity.

(2) $\hat{E}(A) = T_1 C_1 + \hat{\alpha}_T \cdot C_2$, which is the estimate of the upper bound on possible values of long-term transmissive capacity, where,

$$\hat{\alpha}_T = \exp\{\hat{\mu}_Y\} \cdot \phi_t(\tfrac{1}{2}\hat{\sigma}_Y^2)$$

and,

$$\phi_t(S) = 1 + \frac{t-1}{t} \cdot S + \frac{(t-1)^3}{t^2(t+1)} \frac{S^2}{2!} + \frac{(t-1)^5}{t^3(t-1)(t+3)} \frac{s^3}{3!} + \ldots\ldots$$

(3) $\hat{E}(H)$ by the Monte Carlo method. This is the estimate of the lower bound on possible values of long-term transmissive capacity.

Other estimators, such as $\hat{\alpha}_G$ and the confidence limits on μ_F, may also be calculated at this step.

STEP 4

Using,

$$Q_{20} = \frac{A_d}{\frac{S_t}{Q} + \frac{\Delta s_L}{Q}(7 - \log t)}$$

calculate values of Q_{20} for,

(1) $\Delta s_L = \frac{264Q}{\exp(\hat{\mu}_F)}$

(2) $\Delta s_L = \frac{264Q}{\hat{E}(A)}$

(3) $\Delta s_L = \frac{264Q}{\hat{E}(H)}$

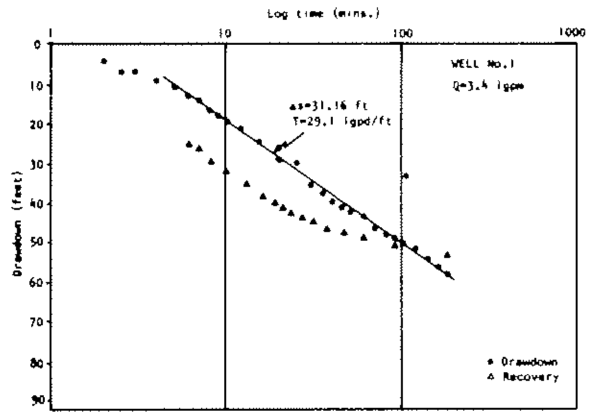
These values are the estimates of the expected value of 20-year safe yield and upper and lower bounds on possible values of 20-year safe yield, respectively.

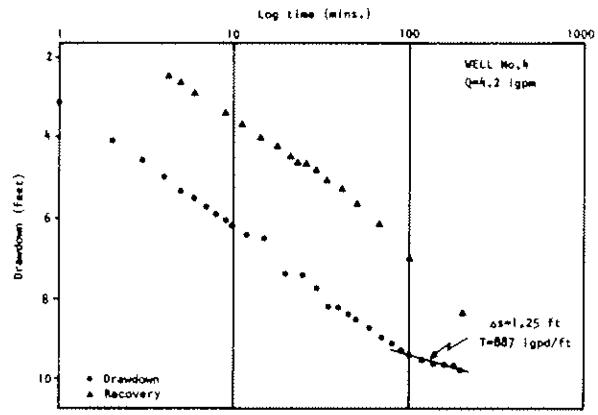
Values of Q_{20} corresponding to estimates of $\hat{\alpha}_G$ and the confidence limits on $\hat{\mu}_F$ may also be calculated at this step.

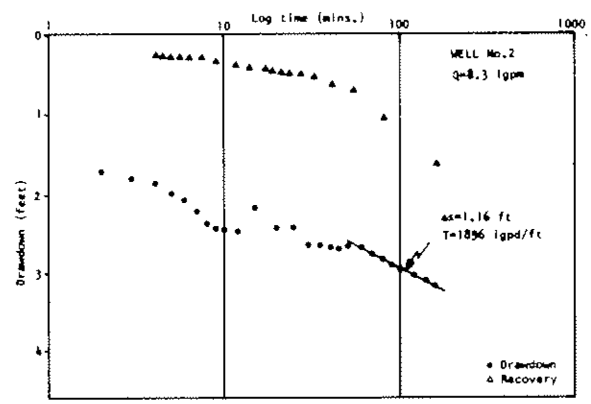
To use the estimates of long-term transmissive capacity to make predictions of drawdowns resulting from arbitrary production programs, refer to appendix 5.

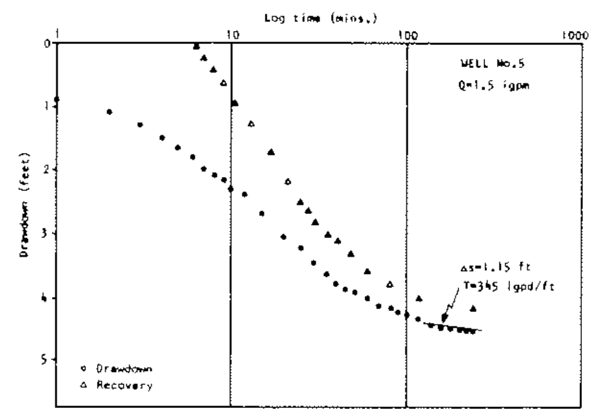
APPENDIX 3.

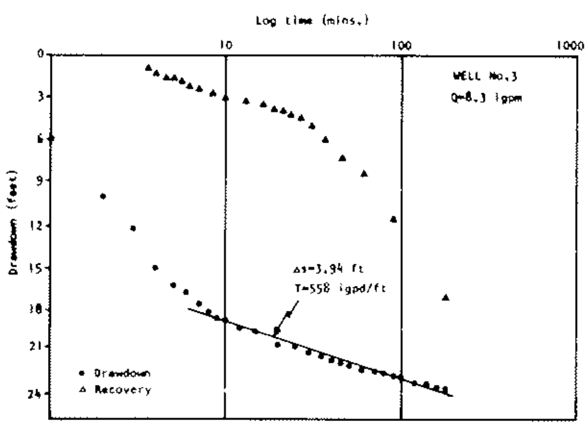
INNISFAIL: SHORT-TERM PUMP TESTS

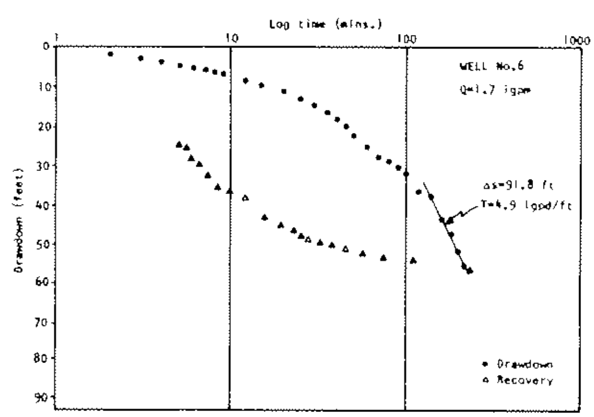


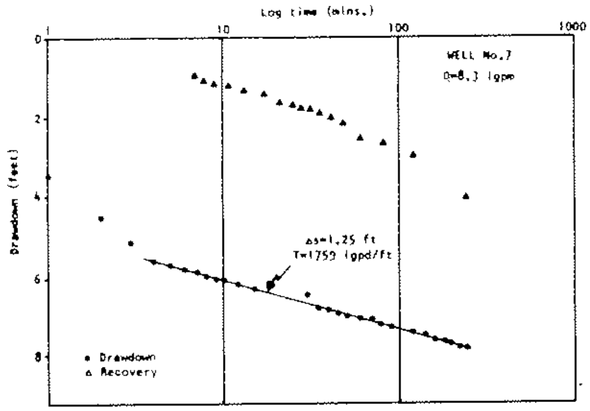


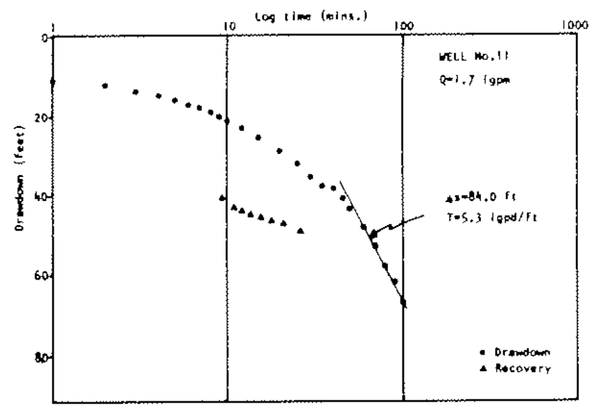


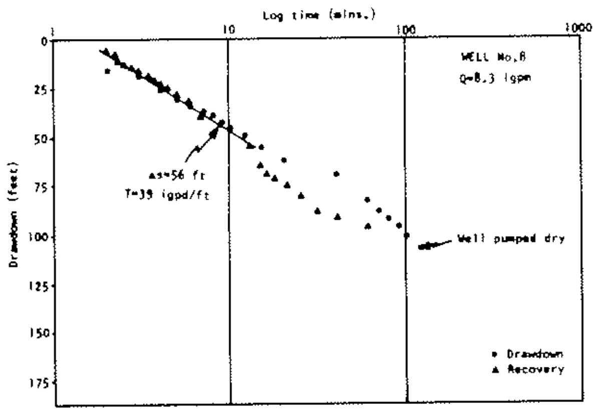


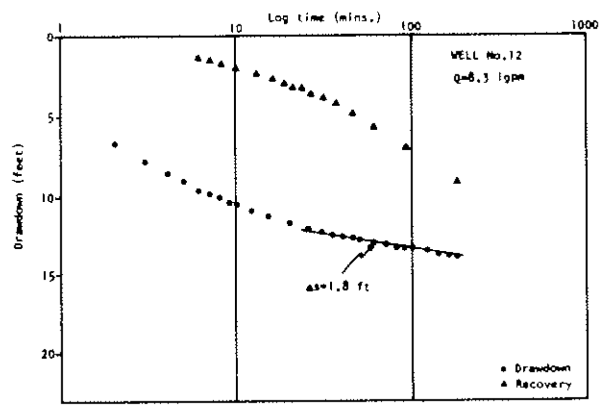


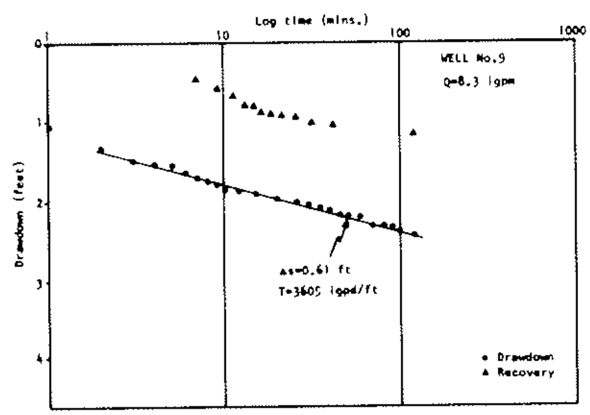


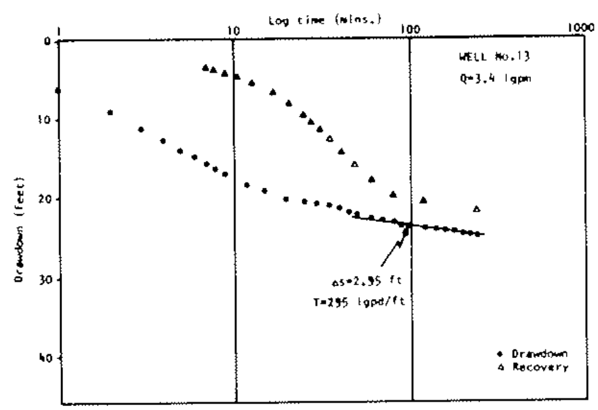

Log time (mins.)
WELL No.7
Q=8.3 Igpm
Drawdown (feet)
Δs=1.25 ft
T=1759 Igpd/ft
• Drawdown
Δ Recovery

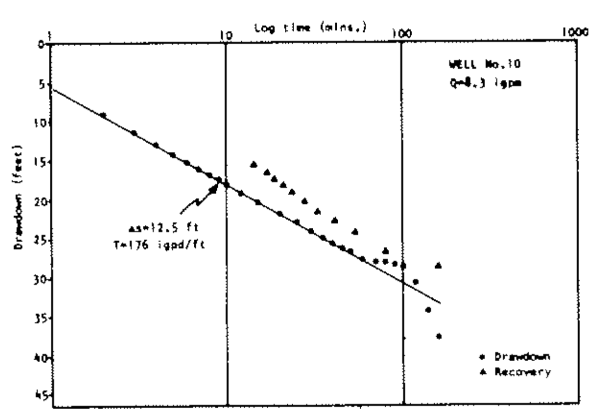

Log time (mins.)
WELL No.11
Q=1.7 Igpm
Drawdown (feet)
Δs=84.0 ft
T=5.3 Igpd/ft
• Drawdown
▲ Recovery

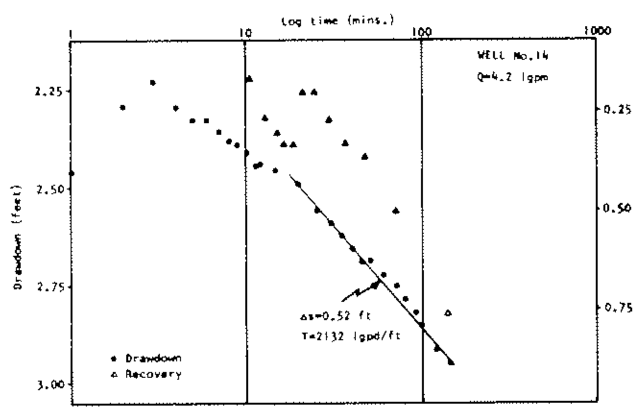

Log time (mins.)
WELL No.8
Q=8.3 Igpm
Drawdown (feet)
Δs=56 ft
T=39 Igpd/ft
Well pumped dry
• Drawdown
▲ Recovery

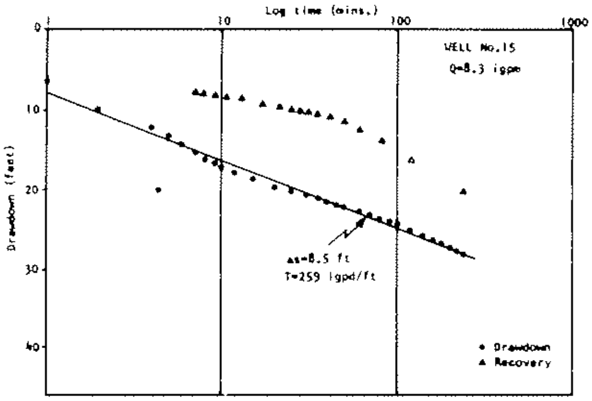

Log time (mins.)
WELL No.12
Q=8.3 Igpm
Drawdown (feet)
Δs=1.8 ft
• Drawdown
▲ Recovery

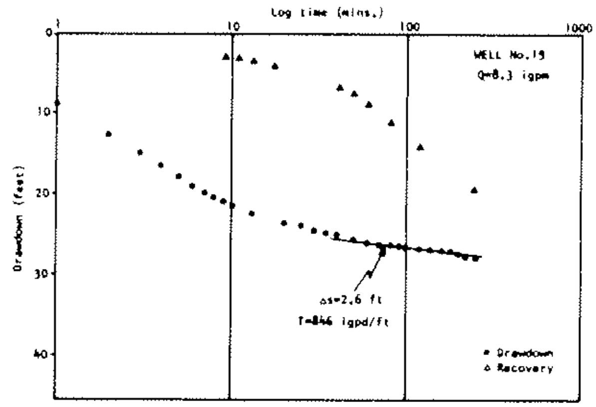

Log time (mins.)
WELL No.9
Q=8.3 Igpm
Drawdown (feet)
Δs=0.61 ft
T=3605 Igpd/ft
• Drawdown
▲ Recovery

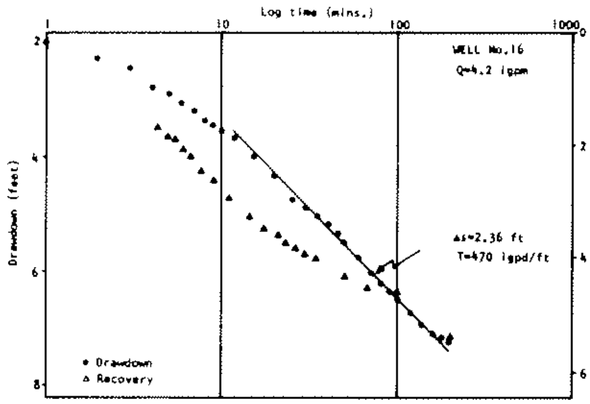

Log time (mins.)
WELL No.13
Q=3.4 Igpm
Drawdown (feet)
Δs=2.95 ft
T=295 Igpd/ft
• Drawdown
Δ Recovery

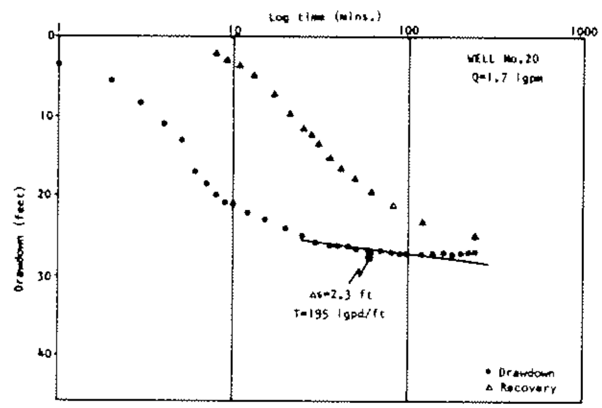

Log time (mins.)
WELL No.10
Q=8.3 Igpm
Drawdown (feet)
Δs=12.5 ft
T=176 Igpd/ft
• Drawdown
▲ Recovery

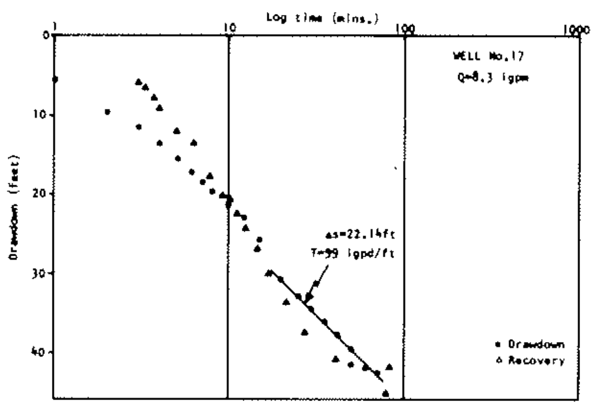

Log time (mins.)
WELL No.14
Q=4.2 Igpm
Drawdown (feet)
Δs=0.52 ft
T=2132 Igpd/ft
• Drawdown
Δ Recovery

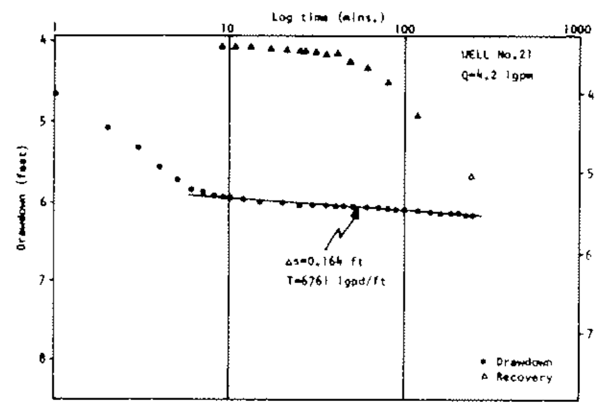

Log time (mins.)
WELL No.15
Q=8.3 igpm
Drawdown (feet)
Δs=8.5 ft
T=259 Igpd/ft
• Drawdown
▲ Recovery

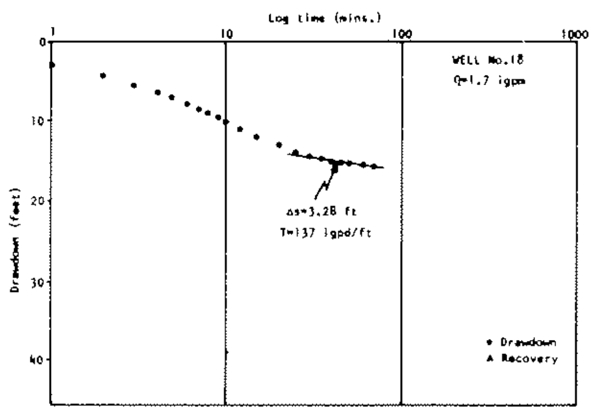

Log time (mins.)
WELL No.19
Q=8.3 igpm
Drawdown (feet)
Δs=2.6 ft
T=846 Igpd/ft
• Drawdown
Δ Recovery

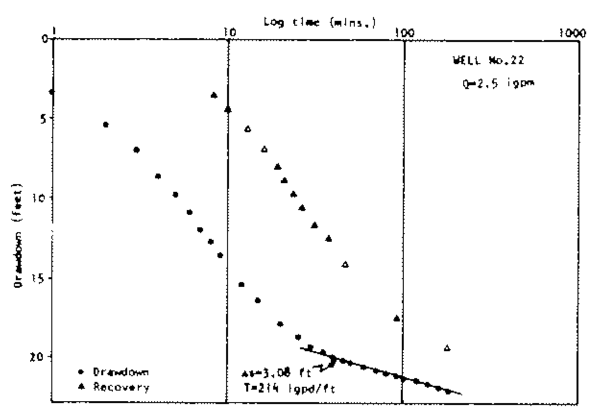

Log time (mins.)
WELL No.16
Q=4.2 igpm
Drawdown (feet)
Δs=2.36 ft
T=470 Igpd/ft
• Drawdown
Δ Recovery

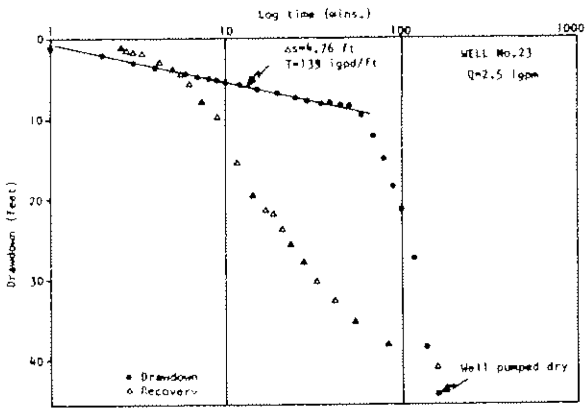

Log time (mins.)
WELL No.20
Q=1.7 igpm
Drawdown (feet)
Δs=2.3 ft
T=195 Igpd/ft
• Drawdown
Δ Recovery

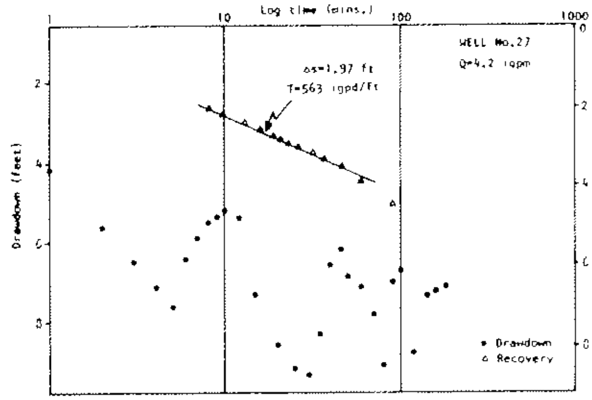

Log time (mins.)
WELL No.17
Q=8.3 igpm
Drawdown (feet)
Δs=22.14ft
T=99 Igpd/ft
• Drawdown
Δ Recovery

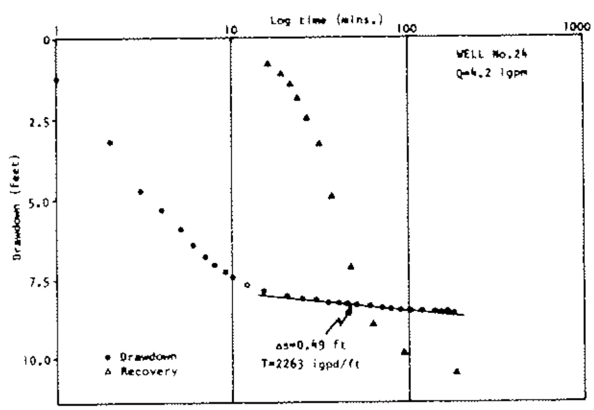

Log time (mins.)
WELL No.21
Q=4.2 igpm
Drawdown (feet)
Δs=0.164 ft
T=6761 Igpd/ft
• Drawdown
Δ Recovery

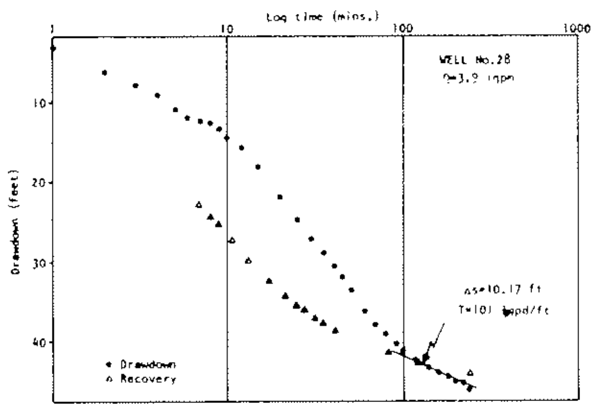

Log time (mins.)
WELL No.18
Q=1.7 igpm
Drawdown (feet)
Δs=3.28 ft
T=137 Igpd/ft
• Drawdown
▲ Recovery

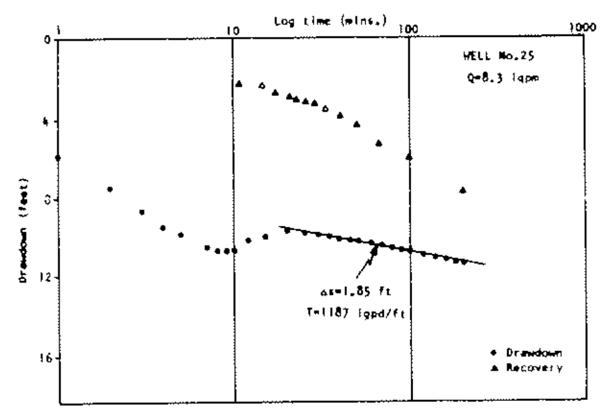

Log time (mins.)
WELL No.22
Q=2.5 igpm
Drawdown (feet)
• Drawdown
▲ Recovery
Δs=3.08 ft
T=214 Igpd/ft

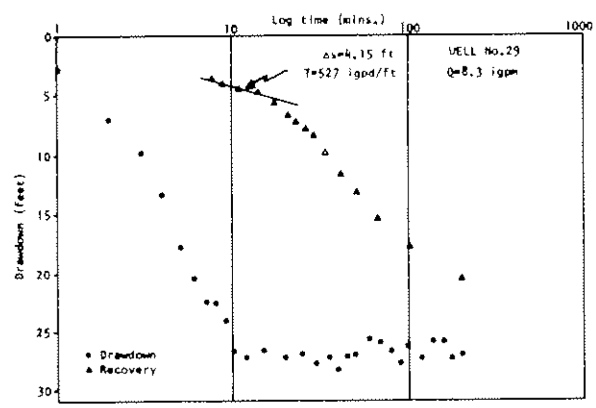

Log time (mins.)
WELL No.23
Q=2.5 igpm
Δs=4.76 ft
T=138 igpd/ft
Well pumped dry
Drawdown (feet)
Drawdown
Recovery

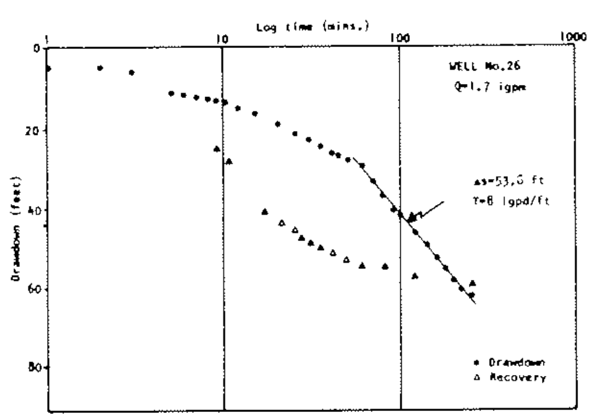

Log time (mins.)
WELL No.27
Q=4.2 igpm
Δs=1.97 ft
T=563 igpd/ft
Drawdown (feet)
Drawdown
Recovery

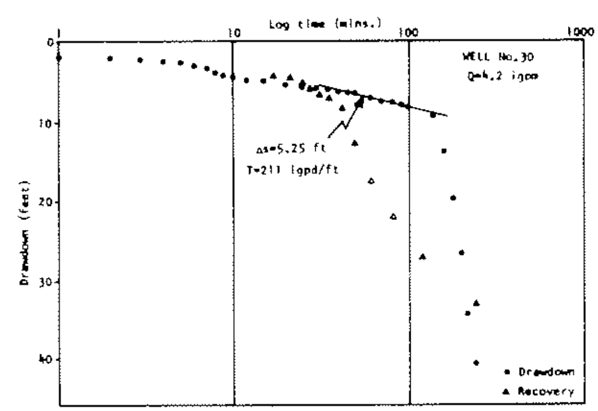

Log time (mins.)
WELL No.24
Q=4.2 igpm
Δs=0.49 ft
T=2263 igpd/ft
Drawdown (feet)
Drawdown
Recovery

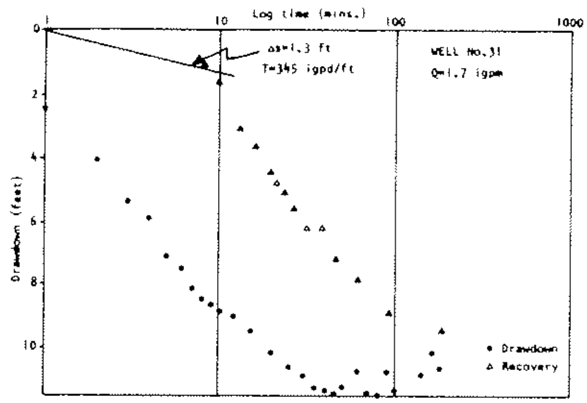

Log time (mins.)
WELL No.28
Q=3.9 igpm
Δs=10.17 ft
T=101 igpd/ft
Drawdown (feet)
Drawdown
Recovery

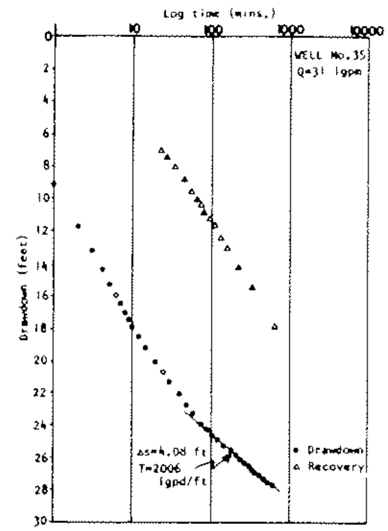

Log time (mins.)
WELL No.25
Q=8.3 igpm
Δs=1.85 ft
T=1187 igpd/ft
Drawdown (feet)
Drawdown
Recovery

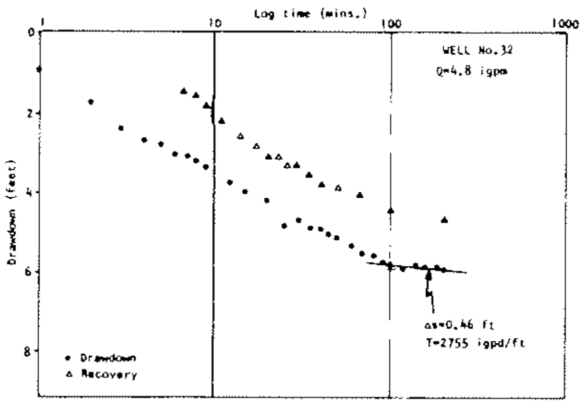

Log time (mins.)
WELL No.29
Q=8.3 igpm
Δs=4.15 ft
T=527 igpd/ft
Drawdown (feet)
Drawdown
Recovery

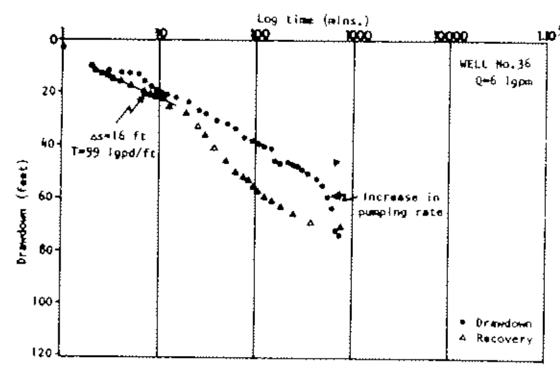

Log time (mins.)
WELL No.26
Q=1.7 igpm
Δs=53.0 ft
T=8 igpd/ft
Drawdown (feet)
Drawdown
Recovery

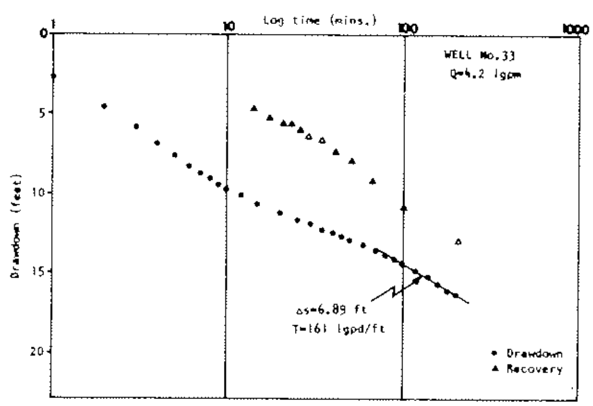

Log time (mins.)
WELL No.30
Q=4.2 igpm
Δs=5.25 ft
T=211 igpd/ft
Drawdown (feet)
Drawdown
Recovery

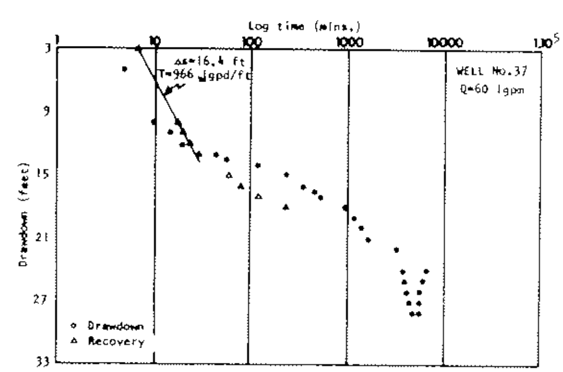

Log time (mins.)
Drawdown (feet)
Δs=1.3 ft
T=345 igpd/ft
WELL No.31
Q=1.7 igpm
• Drawdown
Δ Recovery

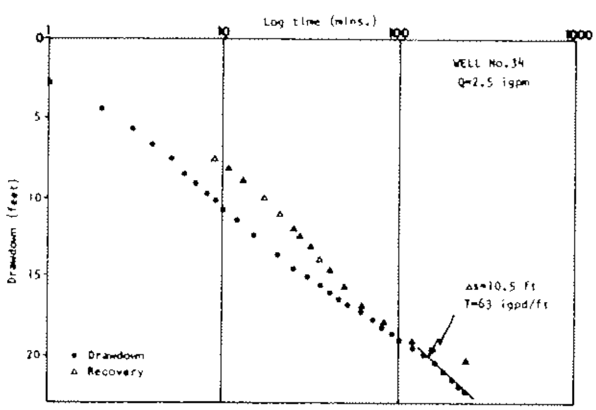

Log time (mins.)
Drawdown (feet)
WELL No.35
Q=31 igpm
Δs=4.08 ft
T=2006 igpd/ft
• Drawdown
Δ Recovery

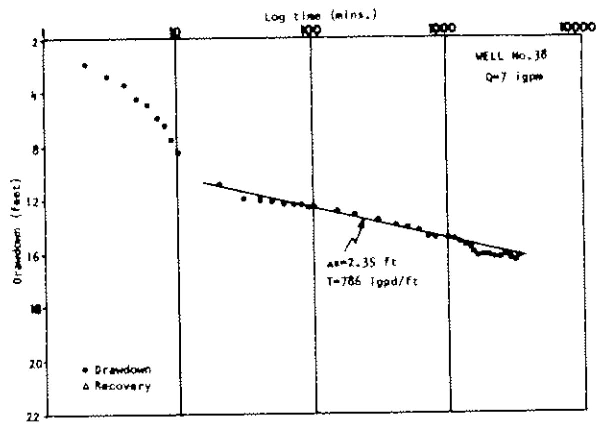

Log time (mins.)
Drawdown (feet)
WELL No.32
Q=4.8 igpm
Δs=0.46 ft
T=2755 igpd/ft
• Drawdown
Δ Recovery

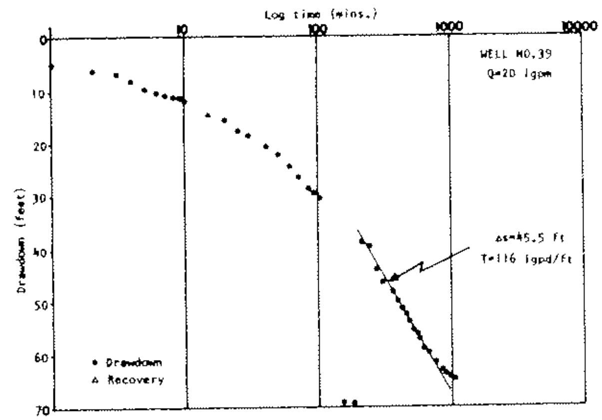

Log time (mins.)
Drawdown (feet)
WELL No.36
Q=6 igpm
Δs=16 ft
T=99 igpd/ft
Increase in pumping rate
• Drawdown
Δ Recovery

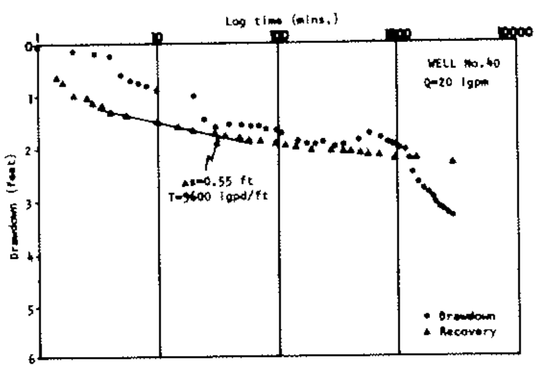

Log time (mins.)
Drawdown (feet)
WELL No.33
Q=4.2 igpm
Δs=6.89 ft
T=161 igpd/ft
• Drawdown
▲ Recovery

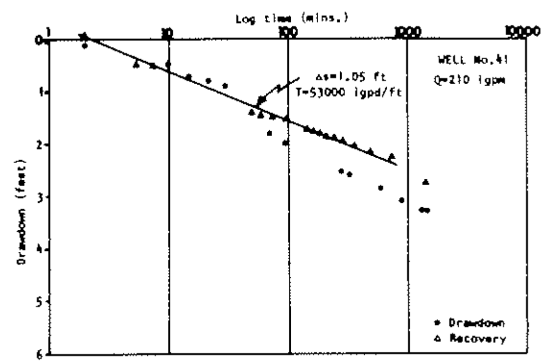

Log time (mins.)
Drawdown (feet)
Δs=16.4 ft
T=966 igpd/ft
WELL No.37
Q=60 igpm
• Drawdown
Δ Recovery


Log time (mins.)
Drawdown (feet)
WELL No.34
Q=2.5 igpm
Δs=10.5 ft
T=63 igpd/ft
• Drawdown
Δ Recovery


Log time (mins.)
WELL No.38
Q=7 igpm
Δs=2.35 ft
T=786 igpd/ft
Drawdown (feet)
Drawdown
Recovery

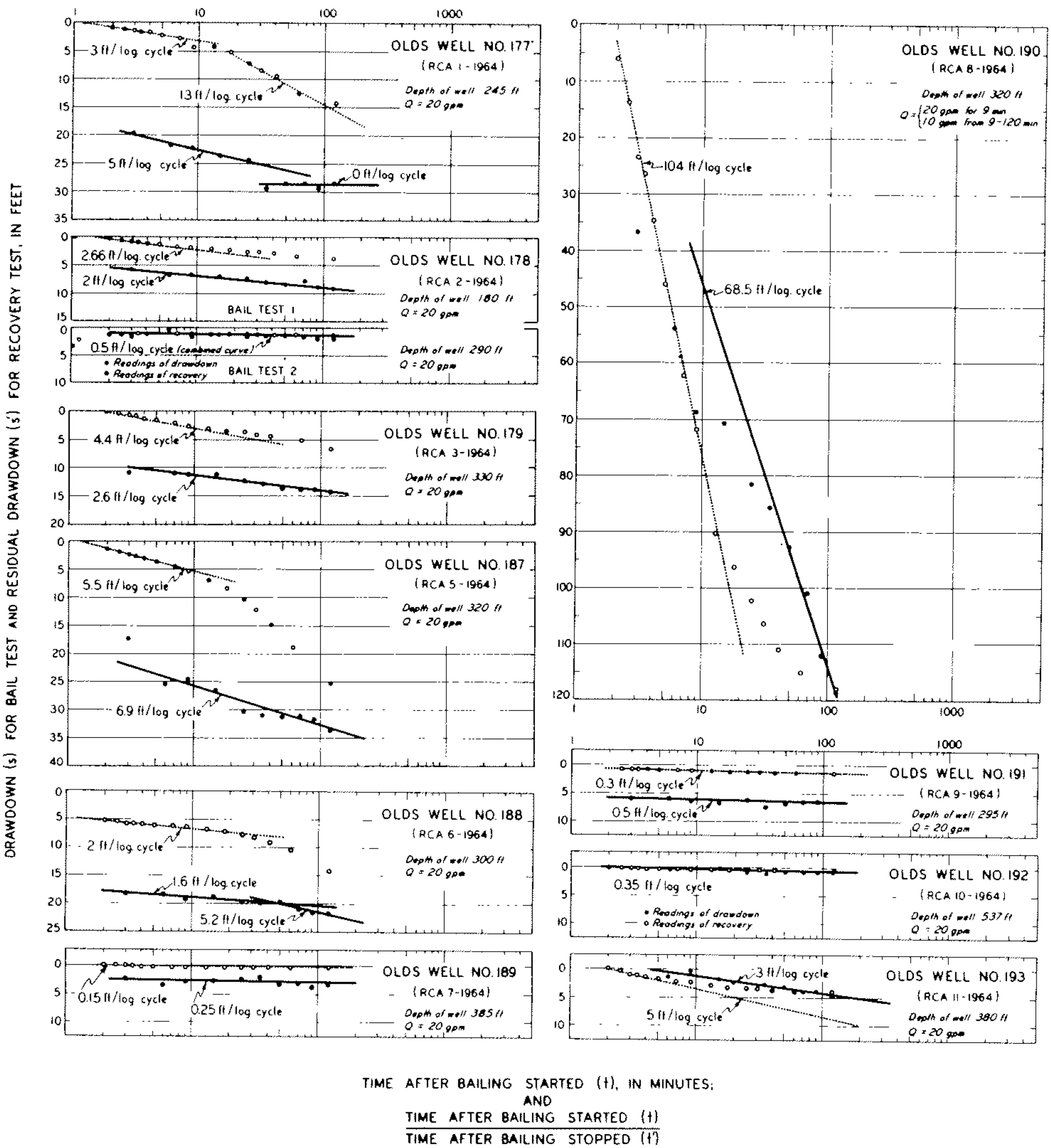

Log time (mins.)
WELL NO.39
Q=20 igpm
Δs=45.5 ft
T=116 igpd/ft
Drawdown (feet)
Drawdown
Recovery

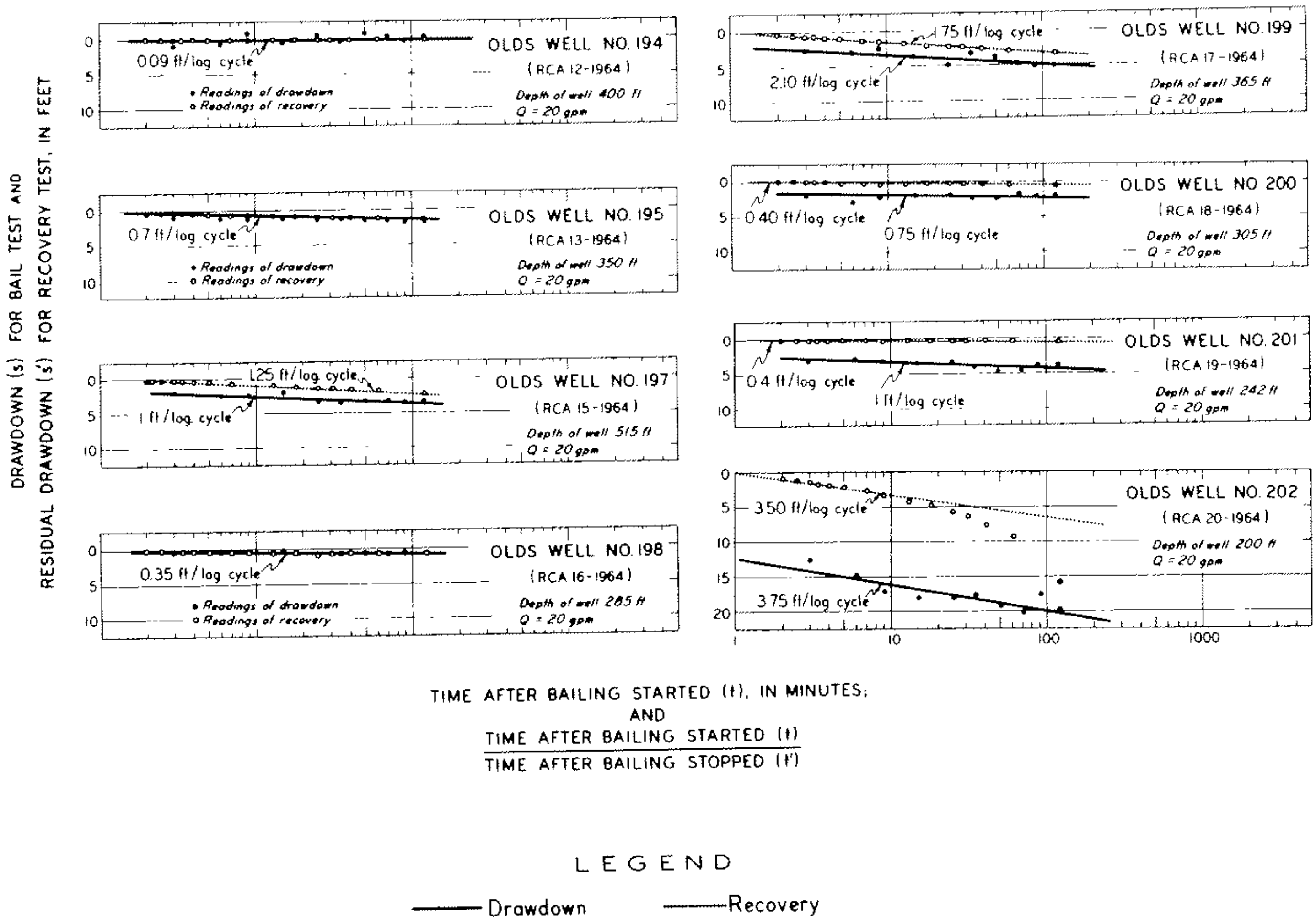

Log time (mins.)
WELL No.40
Q=20 igpm
Δs=0.55 ft
T=9600 igpd/ft
Drawdown (feet)
Drawdown
Recovery

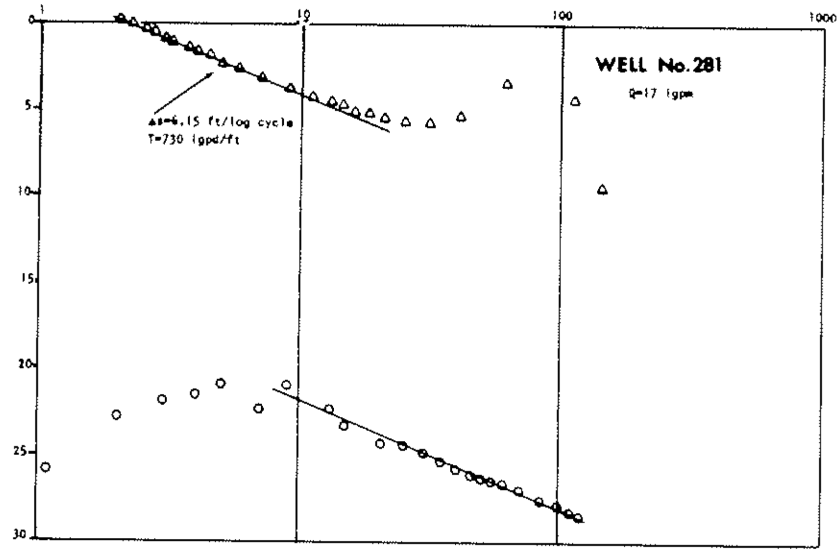

Log time (mins.)
WELL No.41
Q=210 igpm
Δs=1.05 ft
T=53000 igpd/ft
Drawdown (feet)
Drawdown
Recovery

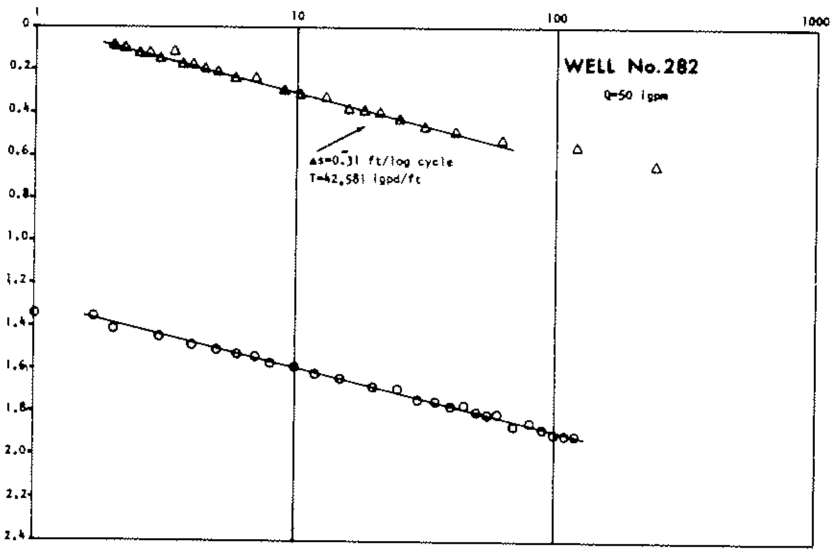
APPENDIX 4.

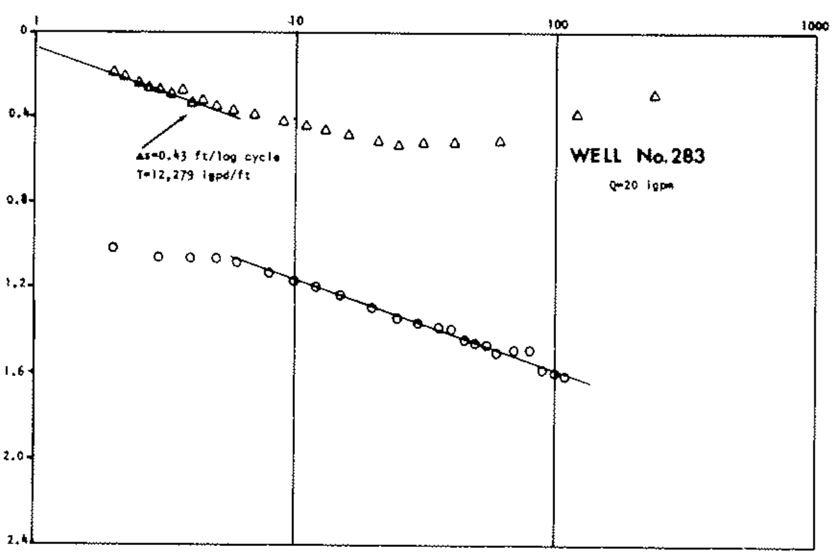
OLDS: SHORT-TERM BAIL AND PUMP TESTS

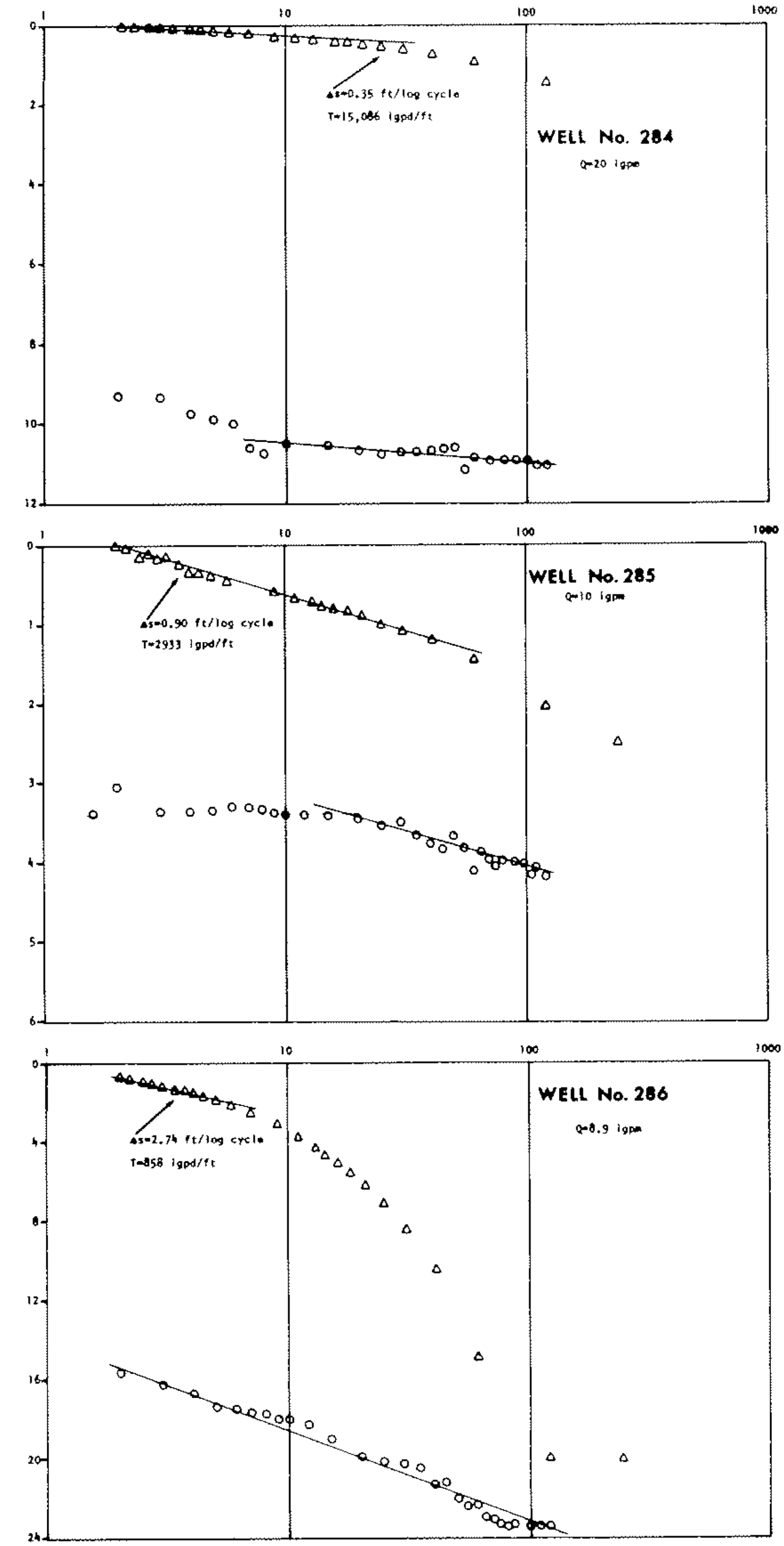


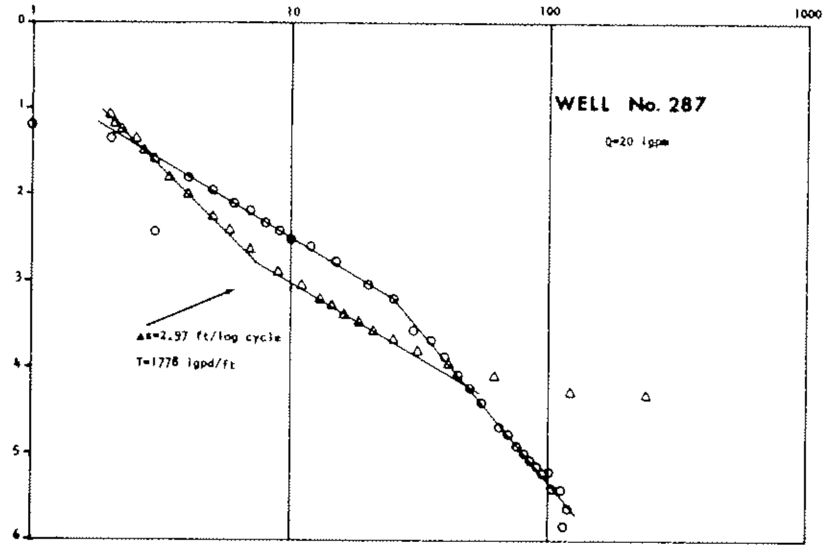

DRAWDOWN (s) FOR BAIL TEST AND
RESIDUAL DRAWDOWN (s') FOR RECOVERY TEST, IN FEET
OLDS WELL NO. 194
(RCA 12-1964)
0.09 ft/log cycle
• Readings of drawdown
o Readings of recovery
Depth of well 400 ft
Q = 20 gpm
OLDS WELL NO. 195
(RCA 13-1964)
0.7 ft/log cycle
• Readings of drawdown
o Readings of recovery
Depth of well 350 ft
Q = 20 gpm
OLDS WELL NO. 197
(RCA 15-1964)
1.25 ft/log cycle
1 ft/log cycle
Depth of well 515 ft
Q = 20 gpm
OLDS WELL NO. 198
(RCA 16-1964)
0.35 ft/log cycle
• Readings of drawdown
o Readings of recovery
Depth of well 285 ft
Q = 20 gpm
OLDS WELL NO. 199
(RCA 17-1964)
1.75 ft/log cycle
2.10 ft/log cycle
Depth of well 365 ft
Q = 20 gpm
OLDS WELL NO. 200
(RCA 18-1964)
0.40 ft/log cycle
0.75 ft/log cycle
Depth of well 305 ft
Q = 20 gpm
OLDS WELL NO. 201
(RCA 19-1964)
0.4 ft/log cycle
1 ft/log cycle
Depth of well 242 ft
Q = 20 gpm
OLDS WELL NO. 202
(RCA 20-1964)
3.50 ft/log cycle
3.75 ft/log cycle
Depth of well 200 ft
Q = 20 gpm
1
10
100
1000
TIME AFTER BAILING STARTED (t), IN MINUTES;
AND
TIME AFTER BAILING STARTED (t) / TIME AFTER BAILING STOPPED (t')
LEGEND
Drawdown
Recovery

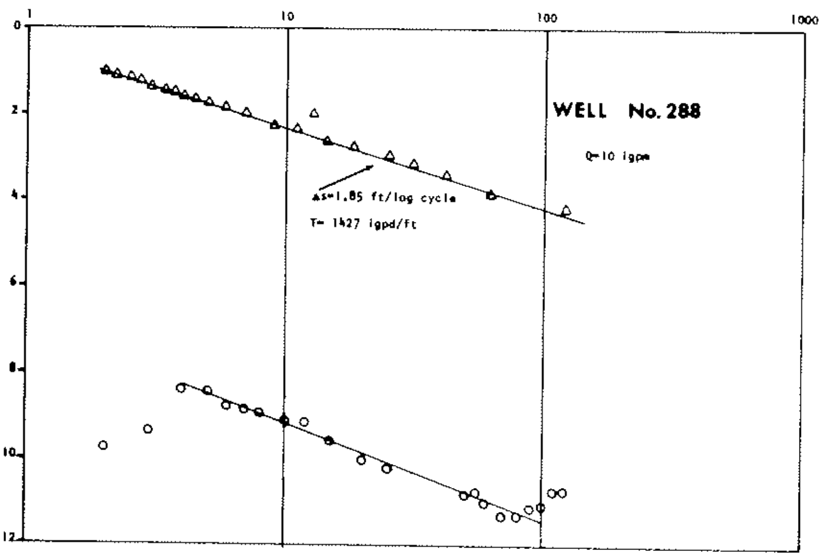

WELL No.281
Q=17 igpm
Δs=6.15 ft/log cycle
T=730 igpd/ft
1
10
100
1000
0
5
10
15
20
25
30

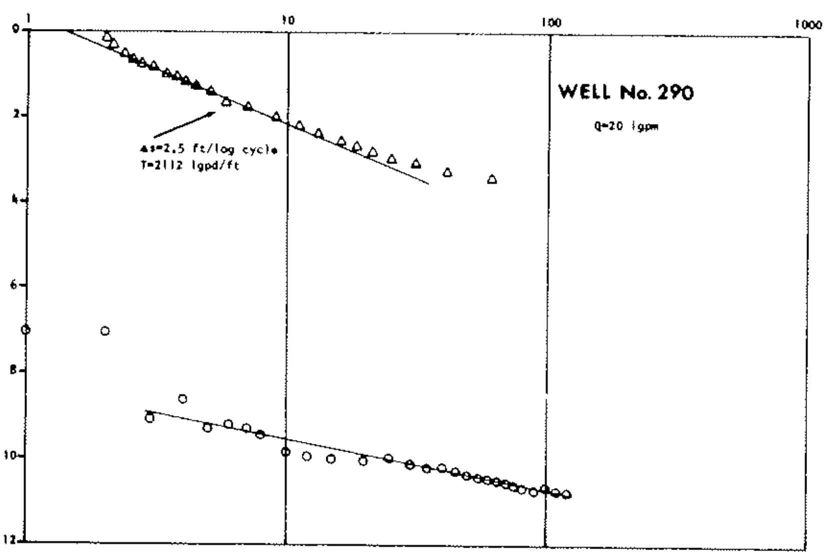

WELL No.282
Q=50 igpm
Δs=0.31 ft/log cycle
T=42,581 igpd/ft
1
10
100
1000
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4

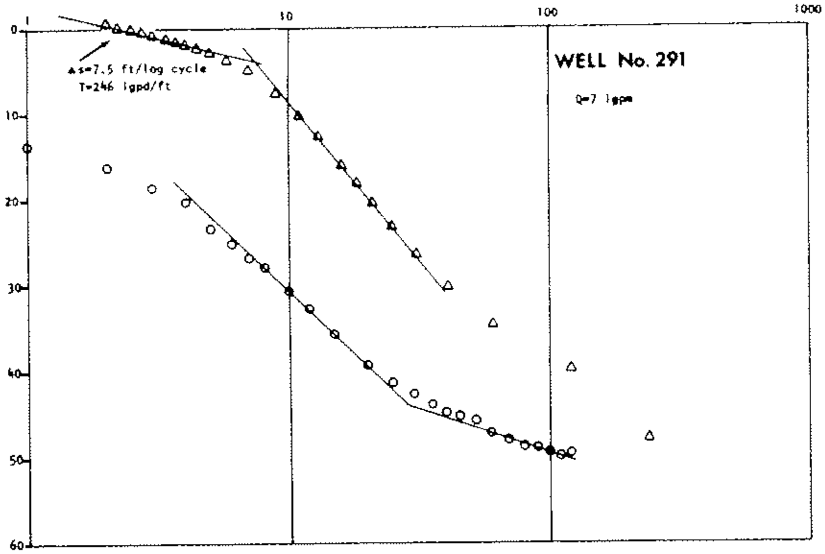

WELL No.283
Q=20 igpm
Δs=0.43 ft/log cycle
T=12,279 igpd/ft
1
10
100
1000
0
0.4
0.8
1.2
1.6
2.0
2.4

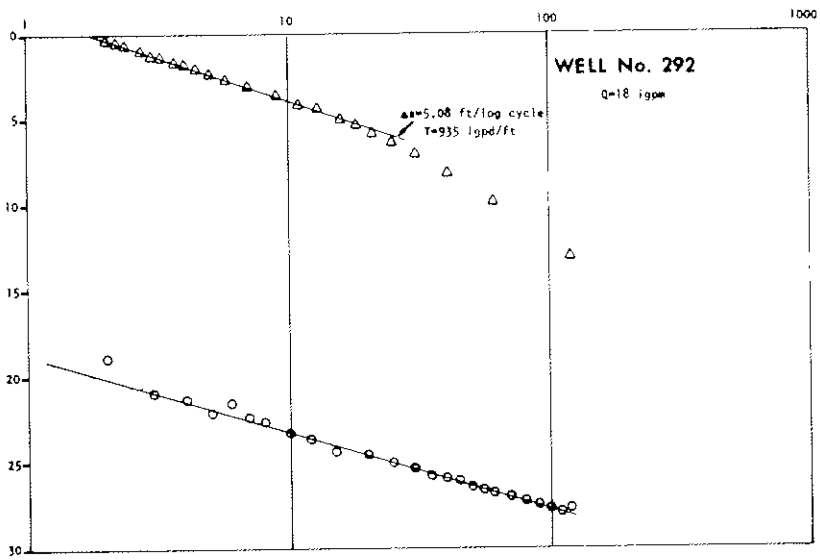

WELL No. 284
Q=20 igpm
Δs=0.35 ft/log cycle
T=15,086 igpd/ft
WELL No. 285
Q=10 igpm
Δs=0.90 ft/log cycle
T=2933 igpd/ft
WELL No. 286
Q=8.9 igpm
Δs=2.74 ft/log cycle
T=858 igpd/ft

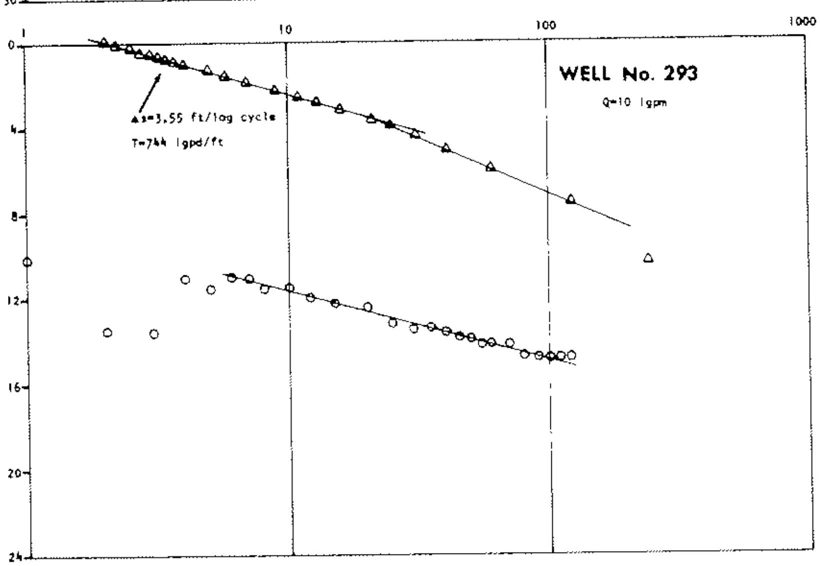

WELL No. 287
Q=20 igpm
Δs=2.97 ft/log cycle
T=1778 igpd/ft
1
10
100
1000
0
1
2
3
4
5
6

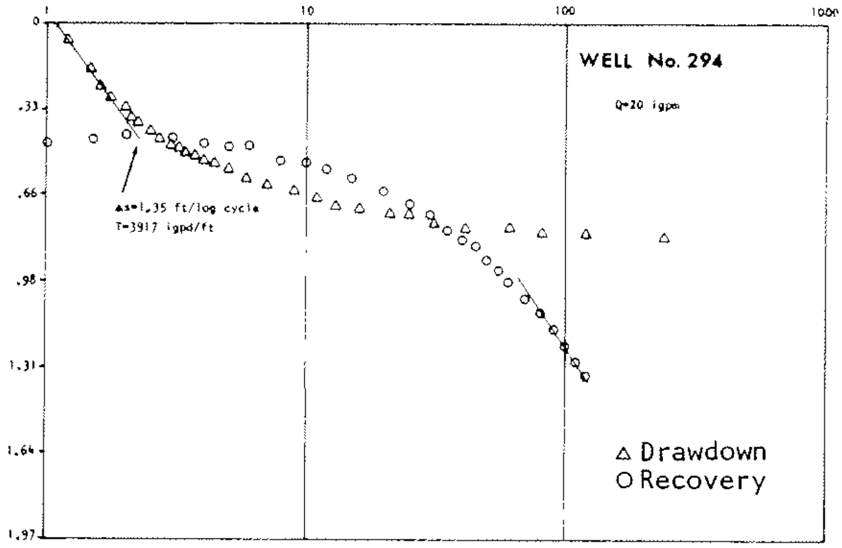

WELL No. 288
Q=10 igpm
Δs=1.85 ft/log cycle
T= 1427 igpd/ft
1
10
100
1000
0
2
4
6
8
10
12

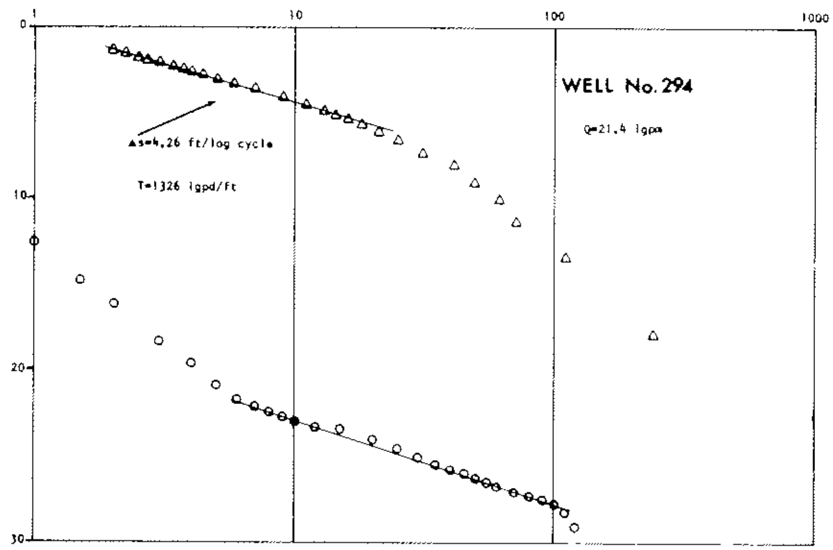

WELL No. 290
Q=20 igpm
Δs=2.5 ft/log cycle
T=2112 igpd/ft
1
10
100
1000
0
2
4
6
8
10
12

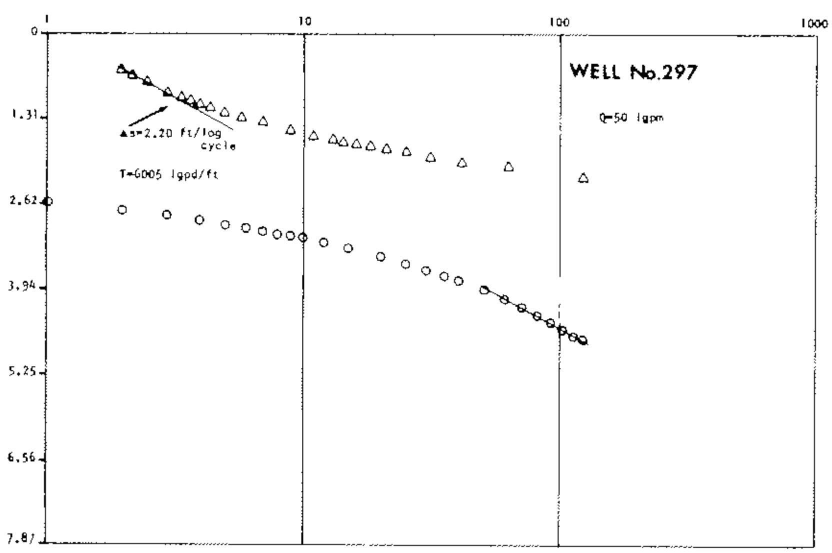

WELL No. 291
Q=7 Igpm
Δs=7.5 ft/log cycle
T=246 Igpd/ft


WELL No. 292
Q=18 igpm
Δs=5.08 ft/log cycle
T=935 Igpd/ft


WELL No. 293
Q=10 Igpm
Δs=3.55 ft/log cycle
T=744 Igpd/ft


1
10
100
1000
0
.33
.66
.98
1.31
1.64
1.97
WELL No. 294
Q=20 igpm
Δs=1.35 ft/log cycle
T=3917 igpd/ft
Δ Drawdown
O Recovery


1
10
100
1000
0
10
20
30
WELL No. 294
Q=21.4 igpm
Δs=4.26 ft/log cycle
T=1326 igpd/ft


1
10
100
1000
0
1.31
2.62
3.94
5.25
6.56
7.87
WELL No.297
Q=50 igpm
Δs=2.20 ft/log cycle
T=6005 igpd/ft

APPENDIX 5

COMPUTER PROGRAM TO CALCULATE FUTURE DRAWDOWNS

The computer program to evaluate the equation given on page 44 for drawdown resulting from an arbitrary production program is given below along with a sample input and output.

The program calls for the following parameters to be specified, in the order given:

NSLOPE, FORMAT(15):
: the number of straight-line segments of time-drawdown curve

(DELS(I),I=1,NSLOPE), FORMAT(16F5.0):
: the slopes of the straight-line segments in feet per log-cycle

QCONST, FORMAT(F10.0):
: the constant pumping rate at which the time-drawdown curve was obtained (igpm)

(TDELS(I),I=1,NSLOPE), FORMAT(8E10.0):
: the time at the end of each straight-line segment (mins)

NQ, FORMAT(15):
: the number of intervals of constant production in the production program

(Q(I),I=1,NQ), FORMAT(8F10.0):
: the constant production rates for each interval (igpm)

(TQ(I),I=1,NQ), FORMAT(8E10.0):
: the time at the end of each production interval (mins). Note that TDELS(NSLOPE) must be greater than TQ(NQ)

A sample input with NSLOPE=3 and NQ=10 is given.

The program calculates the drawdown at every (TQ(I),I=1,NQ). The output corresponding to the sample input is also given.

```
************************************** INPUT DATA **************************************

    3
 1.0  2.0  3.0
   100.0
   100.0     1000.0 10000000.
   10
    80.0      115.0     135.0     102.0       0.0      93.0     109.0     141.0
   123.0       97.0
4.320E+04 8.640E+04 1.022E+05 1.454E+05 1.462E+05 1.894E+05 2.326E+05 2.758E+05
3.190E+05 3.622E+05

************************************** OUTPUT DATA **************************************

NUMBER OF SLOPES =    3
                     SLOPE NO     SLOPE(FT/LOG CYCLE)     TIME TC SLOPE(MINS)
                         1                1.00               0.1000E+03
                         2                2.00               0.1000E+04
                         3                3.00               0.1000E+08

 CONSTANT PUMPING TEST RATE =     100.00 (IGPM)
NUMBER OF CONSTANT Q INTERVALS =    10
                     INTERVAL NUMBER          Q(IGPM)           TIME TO INTERVAL(MINS)
                            1                  80.00                 0.4320E+05
                            2                 115.00                 0.8640E+05
                            3                 135.00                 0.1022E+06
                            4                 102.00                 0.1454E+06
                            5                   0.0                  0.1462E+06
                            6                  93.00                 0.1894E+06
                            7                 109.00                 0.2326E+06
                            8                 141.00                 0.2758E+06
                            9                 123.00                 0.3190E+06
                           10                  97.00                 0.3622E+06

   DRAWDOWN(FT)      TIME(MINS)
 1       7.1252   0.4320E+05
 2      10.9649   0.8640E+05
 3      12.8013   0.1022E+06
 4      10.8235   0.1454E+C6
 5       6.9461   0.1462E+06
 6      10.2795   0.1894E+06
 7      11.8859   0.2326E+06
 8      15.0526   0.2758E+C6
 9      13.9786   0.3190E+06
10      11.8695   0.3622E+06
```

```
      COMMON /A/ NSLOPE,QCONST,NQ
      COMMON /B/ DELS(5) ,TDELS(5) ,Q(500),TQ(500)
      COMMON /C/ NSLTQ(500)
      COMMON /D/ NINTSQ(500,5)
      COMMON /E/ S(500)
      CALL INPUT
      CALL SQSORT
      CALL INT
      CALL DRADOW
      STOP
      END
      SUBROUTINE INPUT
      COMMON /A/ NSLOPE,QCONST,NQ
      COMMON /B/ DELS(5) ,TDELS(5) ,Q(500),TQ(500)
      WRITE(6,1)
    1 FORMAT(' NO OF SLOPES  I5')
      READ(5,2) NSLOPE
    2 FORMAT(I5)
      WRITE(6,3)
    3 FORMAT(' SLOPE VALUES (FT/LOG-CYCLE)         16F5.0 ')
      READ(5,4) (DELS(I),I=1,NSLOPE)
    4 FORMAT(16F5.0)
      WRITE(6,5)
    5 FORMAT(' CONSTANT PUMPING TEST RATE (IGPM)    F10.0 ')
      READ(5,6) QCONST
    6 FORMAT(8F10.0)
      WRITE(6,7)
    7 FORMAT(' TIMES TO SLOPE CHANGES (MINS) MAX TDELS>MAX TQ 8E10.0 ')
      READ(5,8) (TDELS(I),I=1,NSLOPE)
    8 FORMAT(8E10.0)
      WRITE(6,9)
    9 FORMAT(' NO OF CONSTANT PRODUCTION INTERVALS   I5 ')
      READ(5,2)  NQ
      WRITE(6,10)
   10 FORMAT(' PRODUCTION RATE   FOR EACH INTERVAL (IGPM)    8F10.0 ')
      READ(5,6)  (Q(I),I=1,NQ)
      WRITE(6,11)
   11 FORMAT(' TIMES TO PRODUCTION INTERVALS (MINS)  8E10.0 ')
      READ(5,8)  (TQ(I),I=1,NQ)
      WRITE(7,12)   NSLOPE
   12 FORMAT('1NUMBER OF SLOPES =',I5)
      WRITE(7,13)
   13 FORMAT(20X,'SLOPE NO',5X,'SLOPE(FT/LOG CYCLE)',5X,'TIME TO SLOPE(M
     &INS)')
      WRITE(7,14)  (I,DELS(I),TDELS(I),I=1,NSLOPE)
   14 FORMAT(22X,I5,10X,F10.2,10X,E12.4)
      WRITE(7,15) QCONST
   15 FORMAT('0 CONSTANT PUMPING TEST RATE =',F10.2,' (IGPM)')
      WRITE(7,16) NQ
   16 FORMAT(' NUMBER OF CONSTANT Q INTERVALS =',I5)
      WRITE(7,17)
   17 FORMAT(20X,'INTERVAL NUMBER',10X,'Q(IGPM)',10X,'TIME TO INTERVAL(M
     &INS)')
      WRITE(7,18)  (I,Q(I),TQ(I),I=1,NQ)
   18 FORMAT(25X,I5,15X,F10.2,10X,E12.4)
      RETURN
      END
      SUBROUTINE SQSORT
      COMMON /A/ NSLOPE,QCONST,NQ
      COMMON /B/ DELS(5) ,TDELS(5) ,Q(500),TQ(500)
      COMMON /C/ NSLTQ(500)
      K = 1
      DO 1 I = 1,NQ
      TIME = TQ(I)
      DO 2 J = K,NSLOPE
      IF(TDELS(J).GE.TIME) GO TO 3
    2 CONTINUE
    3 K = J
    1 NSLTQ(I) = K
      RETURN
      END
```

```
      SUBROUTINE INT
      COMMON /A/ NSLOPE,QCONST,NQ
      COMMON /B/ DELS(5) ,TDELS(5) ,Q(500),TQ(500)
      COMMON /C/ NSLTQ(500)
      COMMON /D/ NINTSQ(500,5)
      DO 2 I = 1,NQ
      NINTSQ(I,1) = 0
      FINALT = TQ(I)
      NSPT = NSLTQ(I)
      IF(NSPT.EQ.1) GO TO 6
      I1 = I-1
      IF(I1.EQ.0) GO TO 6
      NSPT1 = NSPT -1
      PENULT = TQ(I1) + TDELS(NSPT1)
      IF(PENULT.LE.FINALT) GO TO 6
      DO 4 J =1,NSPT1
      PT = TDELS(NSPT -J)
      DO 5 L = 1,I1
      P = PT + TQ(L)
      IF(P.GT.FINALT) GO TO 7
    5 CONTINUE
      L = L+1
    7 NINTSQ(I,J+1) = L
    4 CONTINUE
      M = L
      IF(L.EQ.I1) M = L + 1
      IF(L.LT.I1) M = I
      NINTSQ(I,J+2) = M
      GO TO 2
    6 N = NSPT + 1
      DO 3 K = 2,N
    3 NINTSQ(I,K) = I
    2 CONTINUE
      RETURN
      END
      SUBROUTINE DRADOW
      COMMON /A/ NSLOPE,QCONST,NQ
      COMMON /B/ DELS(5),TDELS(5),Q(500),TQ(500)
      COMMON /C/ NSLTQ(500)
      COMMON /D/ NINTSQ(500,5)
      COMMON /E/ S(500)
      DO 99 I =1,NQ
      NSTF = NSLTQ(I)
      TIME = TQ(I)
      SSS = 0.0
      DO 88 J = 1,NSTF
      N1 = NINTSQ(I,J)
      N2 = NINTSQ(I,J+1)
      IF(N1.EQ.N2) GO TO 88
      N1 = N1 + 1
      SS = 0.0
      DO 77 K = N1,N2
      SUM = 0.0
      M = NSTF - J
      IF(M.EQ.0) GO TO 1
      DO 66 L = 1,M
      A = 1.0
      IF(L.GT.1) A = TDELS(L-1)
   66 SUM = SUM  + DELS(L) * ALOG10(TDELS(L)/A)
    1 A = 1.0
      IF(M.NE.0) A = TDELS(M)
      B = 0.0
      IF(K.NE.1) B = TQ(K-1)
      SUM = SUM + DELS(NSTF+1-J) * ALOG10((TIME-B)/A)
      A = 0.0
      IF(K.NE.1) A = Q(K-1)
   77 SS = SS + SUM * (Q(K) - A) / QCONST
      SSS = SSS + SS
   88 CONTINUE
   99 S(I) = SSS
      WRITE(7,2)
    2 FORMAT('0    DRAWDOWN(FT)',5X,'TIME(MINS)')
      WRITE(7,3) (I,S(I),TQ(I),I=1,NQ)
    3 FORMAT(I4,F12.4,F12.4)
      RETURN
      END
```